Abstract

We study the effects of two electoral rules, single ballot versus double ballot rule, in determining 1) coalition formation across parties (or candidates) and 2) policy determination. We show that with an highly polarized electorate and small extremist parties, a double ballot system may be an effective way to reduce the influence of extremists (parties and voters) on policy. This holds even if ex post renegotiation across candidates between the two ballots is allowed, and under some conditions, even if extremist parties are larger than moderate ones. We test the theory on Italian municipal elections data, exploiting a natural experiment provided by the Italian institutional setting. Mayors in municipalities with less than 15,000 inhabitants are elected according a single ballot rule, Mayors in municipalities above the 15,000 inhabitants according a double ballot rule. Results are consistent with theory.

Moderating Political Extremism. Single ballot versus two ballot elections

Massimo Bordignon (Catholic University of Milan), Guido Tabellini (Boccon

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1 Introduction

In many electoral systems, politicians are not directly elected following the results of the first round of elections, but rather the results of a first ballot are just used to select a subset of the original candidates over which citizens are asked to vote again at a second ballot. The French Presidential electoral system -where the two candidates who get most votes at the first round run again at a second one to determine the final winner- is probably the most universally known example. But variants of this *dual ballot system* (or *run off system*) can be found in many countries, for both legislative and executives elections, and for both local and national elections (see Cox 1997, chapter). One obvious question is which difference does this system make, if any, with respect to the most commonly used *single ballot system* where candidates or parties are instead directly elected at the first round. In particular, one would like to know whether the dual ballot system makes a difference in terms of the number and composition of parties (or coalitions of parties) which run at the election, and secondly, and most importantly, whether it makes a difference in terms of the policies which are then enacted. In spite of its obvious policy relevance, remarkably little research has been devoted to this issue. The few studies which do exist on the dual ballot system have mainly focussed on the first question, wondering how the well known Duverger's Law and Hypothesis –predicting number of parties and candidates as a result of the electoral system– would extend to a dual ballot situation (Sartori, Fisichella Cox, 1997; Callander,2005, Messner and Polborn, 2004;

see section V for further discussion of the literature). Furthermore, to the best of our knowledge, no one has attempted to test empirically if a dual ballot system makes a clear difference in terms of policy enacted, as a result of the different coalitions or parties which form with respect to a single ballot one. This is what we plan to do in the present paper, by exploiting a natural experiment which is offered by the Italian case.

To this aim we build a simple theoretical model, derive a number of clear-cut predictions and test them against Italian data on municipal elections. Our theoretical model suggests that with a highly ideologically polarized electorate, a double ballot system may be an effective way to reduce the influence of extremist parties and voters on policy. A dual electoral system reduces the bargaining powers of the smaller and usually more extreme parties, because some extremist voters would prefer to vote on the surviving candidates at the second turn rather than abstain and risk that the less desired candidate gets elected at the final elections. In the pre-electoral bargaining stage, when parties bargain over policies and coalitions, this leads them to agree on policy platforms which are closer to the preferences of the moderate votes and candidates. Interestingly, we also show that this basic prediction of the model holds even if renegotiation among parties is allowed between the two ballots, and even if , contrary to most empirical evidence, extremist parties are the larger rather than the smaller parties in the coalitions.

To test these predictions, we build upon a natural experiment which is offered by the Italian case. In Italy, mayors and councils in municipalities below 15,000 inhabitants are elected according to a single ballot rule; the candidate who gets more votes is directly elected as mayor and her supporting coalition of parties gets a neat majority (2/3 of seats) in the municipal council regardless the actual votes collected at the elections (because of a "majority prize"). On the contrary, for municipalities above 15,000 inhabitants, a dual ballot system is in place. Unless one candidate gets more than 50% of the votes at the first ballot, in which case she/he is directly elected, the two most voted candidates at the first round run again at a second ballot to determine the final winner. The most voted candidate at the second round becomes mayor and again, in most cases (see section VI below), a "majority prize" guarantees her a large majority in the municipality council. Parties share seats in the municipality council on the basis of the results of the first round elections, with supporting parties of the winning candidate sharing at least 60% of seats. Importantly, a mayor, once elected, cannot be substituted by her/his council; in case of a no confiance vote by the council, new elections take place. On the basis of a mutual agreement, defeated first round candidates can endorse one of the two surviving candidates at the second turn, thus sharing the majority prize in case the endorsed candidate is elected at the final ballot. But policy platforms cannot be changed between rounds; defeated first round candidates who decide to support a surviving candidate are asked to sign the latter's policy platform, which is presented to voters before the first round of voting.

Our preliminary empirical results support the predictions of our theory. We first show that voters are mobile between ballots, a necessary condition for our theoretical analysis. We then show that our predictions about number of candidates and coalition formation in the two electoral systems are supported by data. We plan to extend the analysis of the remaining predictions of the model in the future.

Our results have also clear-cut policy implications, at least for the Italian case. Following the 1994 reform, which modified the previously pure proportional system for national elections in a single ballot majority system (for 75% of the parliamentary seats), Italy has moved to a simpler two-coalitions system, alternating in office and mostly able to last for the prescribed 5 years span of the legislation[1]. This was a neat improvement with respect to the previous system, characterized by ever changing governments and legislatures (the average life of an Italian government in the aftermath of the second world war was 10 months) and the same centrist parties continuously in power. But the two coalitions have remained largely heterogeneous and small and extremist parties, pivotal to win the elections, have played a disproportionate role in determining the coalitions' policy and in blocking the necessary economic reforms. Our results suggest that an extension of the municipalities dual ballot system to national elections could offer an effective way to solve these problems.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 studies coalition formation and policy determination under single ballot and dual ballot electoral rules, deriving our basic theoretical predictions. Section 4 extends the model to the case where renegotiation among the two ballots is allowed, basically modelling a stylized version of the Italian rules. Some extensions of the basic model are presented in Section

[1] In 2005 the electoral system has been changed again, with only the support of the ruling majority of the time, by reintroducing a proportional system and by accompanying it with a majority prize for the winning coalitions.

5. Section 6 presents our data set and describe in larger detail the Italian municipality electoral system. Section 7 takes our predictions to data and performs our empirical analysis. Section 8 concludes by summarizing results and suggesting further avenues for research. The Appendix contains formal proofs of the theoretical results.

2 The Model

2.1 Voters

The electorate consists of four groups of voters indexed by $J = 1, 2, 3, 4$, with policy preferences:

$$U^J = -\left|t^J - q\right|$$

where $q \in [0, 1]$ denotes the policy and t^J is group J's bliss point. Thus, voters lose utility at a constant rate if policy is further from their bliss point. The bliss points of each group have a symmetric distribution on the unit interval, with: $t^1 = 0$, $t^2 = \frac{1}{2} - \lambda$, $t^3 = \frac{1}{2} + \lambda$, $t^4 = 1$, and $\frac{1}{2} \geq \lambda > \frac{1}{6}$. Groups 1 and 4 will be called "extremist", groups 2 and 3 "moderate". The assumption $\lambda > \frac{1}{6}$ implies that the electorate is polarized, in the sense that each moderate group is closer to one of the two extremists than to the other moderate group. We briefly discuss the effects of relaxing this assumption in section 5 below.

The two extremist groups have a fixed size $\underline{\alpha}$. The size of the two moderate groups is random: group 2 has size $\overline{\alpha} + \eta$, group 3 has size $\overline{\alpha} - \eta$, where $\overline{\alpha}$ is a known parameter with $\overline{\alpha} > \underline{\alpha}$, and η is a random variable with mean and median equal to 0 and a known symmetric distribution over the interval $[-e, e]$, with $e > 0$. Thus, the two moderate groups have expected size $\overline{\alpha}$, but the shock η shifts voters from one moderate group to the other. We normalize total population size to unity, so that $\overline{\alpha} + \underline{\alpha} = \frac{1}{2}$.

The only role of the shock η is to create some uncertainty about which of the two moderate groups is largest. Specifically, throughout we assume:

$$(\overline{\alpha} - \underline{\alpha}) > e \tag{A1}$$

$$\underline{\alpha}/2 > e \tag{A2}$$

Assumption (A1) implies that, for any realization of the shock η, any moderate group is always larger than any extreme group. Assumption (A2) implies that, for any realization of the shock η, the size of any moderate group is always smaller than the size of the other moderate group plus one of the extreme groups. Again, we discuss the effects of relaxing these assumptions in Section 5. The realization of η becomes known at the election and can be interpreted as a shock to the participation rate.

Figure 1 illustrates the distribution of policy preferences and of group size, for $\eta = 0$.

Finally, throughout we assume that voters vote *sincerely* for the party or coalition that promises to deliver them higher utility. We briefly discuss strategic voting in section 5.

2.2 Candidates

There are four political candidates, $P = 1, 2, 3, 4$, who care about being in government but also have ideological policy preferences corresponding to those of voters:

$$V^P(q, r) = -\sigma \left| t^P - q \right| + E(r) \tag{1}$$

where $\sigma > 0$ is the relative weight on policy preferences, and $E(r)$ are the expected rents from being in government. The ideological policy preferences of each candidate are identical to those of the corresponding group of voters: $t^P = t^J$ for $P = J$. Rents only accrue to the party in government, and are split in proportion to the number of party members. Thus, $r = 0$ for a candidate out of government, $r = R$ if a candidate is in government alone, $r = R/2$ if two candidates have joined to form a two-member party and won the elections (as discussed below, we rule out parties formed by more than two party members). The value of being in government, $R > 0$, is a fixed parameter.

2.3 Policy choice and party formation

Before the election, candidates may merge into parties (or coalitions) and present their policy platforms. We will speak of mergers between candidates as "parties", although they can be thought of as electoral cartels or coalitions of pre-existing parties. Once elected, the governing party cannot be dissolved until the next election, in line with the Italian case.

If a candidate runs alone, he can only promise to voters that he will implement his bliss point: $q^P = t^P$. If a party is formed, then the party can promise to deliver any policy lying in between the bliss points of its party members; thus, a party formed by candidates P and $P\prime$ can offer any $q^{PP'} \in [t^P, t^{P'}]$. But policies outside this interval cannot be promised by this coalition. This assumption can be justified as reflecting lack of commitment by the candidates. A coalition of two candidates can credibly commit to any $q^{PP'} \in [t^P, t^{P'}]$ by announcing the policy platform and the cabinet formation ahead of the election; to credibly move its policy platform towards t^P, the coalition can tilt the cabinet towards party member P. But announcements to implement policies outside of the interval $[t^P, t^{P'}]$ would not be ex-post optimal for any party member and would not be believed by voters. This assumption, also made by Morelli (2004), fits well with the actual rules for electing municipal governments in Italy, where parties have to announce their policy platform ahead of the elections. Once elected, a mayor has the exclusive right to choose and dismiss the cabinet members and if he is forced to step down by his majority (trough a negative confiance vote in the municipality council), new elections follow automatically. Hence, the council cannot choose a new mayor in between the elections, while the mayor can always threaten his majority of an earlier dismissal by resigning from office. This is an effective way to keep the coalition together and adds to the commitment powers of the candidates ahead of elections [2].

We assume that parties can contain at most two members, and these members have to be adjacent candidates[3]. Thus, say, candidate 2 can form a party with either candidate 3 or candidate 1, while candidate 1 can only form a party with candidate 2. This simplifying assumption also captures a realistic feature. It implies that coalitions are more likely to form between ideologically closer parties, and that moderate parties can sometimes run together, while opposite extremists cannot form a coalition between them, as voters would not support this coalition. This gives moderate candidates an advantage; we model this below by adopting a two stages bargaining model, where moderates are the first movers. Candidates can bargain only over the policy q that will be implemented if they are in government. As we already

[2] Indeed, in our sample, the rate of earlier dismission by municipal governments dramatically fell by as a result of the 1993 electoral reform.

[3] See again Morelli (2004) for a similar modelling choice and Axelrod (1970) for a justification of this assumption.

said, rents from office are fixed and are split equally amongst party members[4]. Bargaining takes place before knowing the realization of the random variable η that determines the relative size of groups 2 and 3, and agreements cannot be renegotiated once the election result is known.

Bargaining takes place according to a two stage process. At the first stage, candidates 2 and 3 bargain with each other to see if they can form a moderate party. Either 2 or 3 is selected with equal probability to be the agenda setter. Whoever is selected (say 2) makes a take-it-or-leave-it offer of a policy q^{23} to the other moderate candidate. If the offer is rejected, the game moves to the second stage. If the offer is accepted, then the moderate party is formed and the two moderate candidates run together at the election. Voters then vote over three alternatives: candidate 1, who would implement $q = t^1$; candidate 4, who would implement $q = t^4$; and the party consisting of candidates $\{2, 3\}$, who would implement $q = q^{23}$. Whoever wins the election then implements its policy and enjoys the rents from office.

At the second stage, the moderate and the extreme candidates simultaneously bargain with each other (1 bargains with 2, while 3 bargains with 4) to see if they can form a moderate-extreme party. In each pair of bargaining candidates, an agenda setter is again randomly selected with equal probabilities. For simplicity, there is perfect correlation: either candidates 1 and 4 are selected as agenda setter, or candidates 2 and 3 are selected. This selection is common knowledge (i.e. all candidates knows who is the agenda setter in the other bargaining pair). The two agenda setters simultaneously choose whether to make a take-it-or-leave-it policy proposal to their potential coalition partner, or to refrain from making any offer. The candidates receiving the offer simultaneously accept it or reject it. If the proposal is accepted, the party is formed and the two candidates run together at the election on the same policy platform. If the proposal is rejected (or if no offer is made), then each candidate in the relevant pair stands alone at the ensuing election, and his policy platform coincides with his bliss point[5]. Again, whoever wins the

[4]If rents were large and wholly contractible at no costs, then each coalition would form at the policy platform that maximizes the probability of winning for the coalition and rents would be used to compensate players and redistribute the expected surplus. But if rents were limited or contractible at some increasingly convex costs, then our results below would still hold qualitatively as coalitions would want to bargain over policies too. The assumption of fixed rents, in addition to simplify the analysis, also captures some institutional features of the Italian example (e.g. the majority prize in the municipality councils for the winning coalition).

[5]Hence, we assume that a candidate (=party) always runs, either alone, or in a coalition

election implements his policy and enjoys the rents from office.

Thus, this second stage can yield one of the following four outcomes. If both proposals are accepted, voters have to choose between two parties $(\{1,2\},\{3,4\})$, each with a known policy platform. If both proposals are rejected (or never formulated), then voters vote over four candidates $(\{1\},\{2\},\{3\},\{4\})$, each running on his bliss point as a policy platform. If one proposal is accepted and the other rejected, then voters cast their ballot over three alternatives: either $(\{1,2\},\{3\},\{4\})$, or $(\{1\},\{2\},\{3,4\})$, depending on who rejects and who accepts. Note that renegotiation is not allowed; that is, if say party $\{1,2\}$ is formed, but 3 and 4 run alone, candidates 1 and 2 are not allowed to renegotiate their common platform.

2.4 Electoral rules

The next sections contrast two electoral rules. Under a single ballot rule, the candidate or the party that wins the relative majority in the single election forms the government. Under a closed dual ballot rule, voters cast two sequential votes. First, they vote on whoever stands for election. The two parties or candidates that obtain more votes are then allowed to compete again in a second round. Whoever wins the second ballot forms the government. We discuss additional specific assumptions about information revelation and renegotiation between the two rounds of election in context, when illustrating in detail the two ballot system. Extensions are considered in section 5.

3 Single ballot

We now derive equilibrium policies and party formation under the single ballot rule. The model is solved by working backwards.

Suppose that the second stage of bargaining is reached. Any candidate running alone (say candidate 1 or 2) does not have a chance of victory if it

with the other candidate (=party). Notice that for the Italian case we study, this makes perfect sense; the electoral system is proportional, and by not running at all, a party would give up the seats that it could conquer in the municipality council even if it did not belong to the winning coalition. More generally, there may be longer term reasons why a party may wish to run even if its candidate cannot win the present elections (e.g. measuring its strength for future alliances).

runs against a moderate-extremist party (say, of candidates $\{3,4\}$ together). The reason is that, by assumption (A2), the size of voters in groups 3 and 4 together is always larger than the size of voters in group 1 or 2 alone, for any shock to the participation rate η. Moreover, given the assumption that $\lambda > 1/6$, voters in the moderate group 3 are ideologically closer to extremist candidate 4 than to the other moderate candidate, 2. Hence, all voters in groups 3 and 4 prefer any policy $q \in [t^3, t^4]$ to the policy t^2. In other words, the party $\{3,4\}$ always gets the support of all voters in groups 3 and 4 for any policy the party might propose, and this is the largest group in a three party equilibrium. This in turn implies that forming a moderate-extremist party is always a dominant strategy for all candidates, irrespective of what the opposing candidates do and of who is the agenda setter inside each party. Hence, we have (a more detailed proof is presented in the appendix):

Proposition 1 *If stage two of bargaining is reached, then the unique equilibrium is a two-party system, where the moderate-extremist parties* $(\{1,2\}, \{3,4\})$ *compete in the elections and have equal chances of winning. The policy platform of each party is the bliss point of whoever happens to be the agenda setter inside each party. Hence, with equal probabilities, the policy actually implemented coincides with the bliss point of any of the four candidates.*

Note that, if all candidates run alone, the extremist candidates do not have a chance. By assumption (A1), the moderate groups are always larger than the extremist groups, for any shock to the participation rate η. Hence, in a four candidates equilibrium, the two moderate candidates win with probability 1/2 each. This means that the moderate candidates 2 and 3 would be better off in the four candidates outcome than in the two-party equilibrium. In both situations, they would win with the same probability, 1/2, but in the former case they would not have to share the rents in case of a victory. But the two moderate candidates are caught in a prisoner's dilemma. In a four candidates situation, each moderate candidate would gain by a unilateral deviation that led him to form a party with his extremist neighbor, since this would guarantee victory at the elections. Hence in equilibrium a two party system always emerges. This in turn reinforces the bargaining power of the extremist candidates. Even if they have no chances of winning on their own, they become an essential player in the coalition. Here we model this by saying that with some probability they are agenda setters and impose their own bliss point on the moderate-extremist coalition. When this happens, the equilibrium policies reflect the policy preferences of extremist candidates, although their voters are a (possibly small) minority. But the result is more general, and would emerge from other bargaining assumptions, as long as the equilibrium policy platforms reflect the bargaining power of both prospective partners.

Next, consider the first stage of the bargaining game. Here, one of the moderate candidates is randomly selected and makes a policy offer to the other moderate candidate. If the offer is accepted, the three parties configuration $(\{1\}, \{2,3\}, \{4\})$ results. If it is rejected, the two-party outcome in stage two described above is reached. Thus, the three party outcome with a centrist party can emerge only if it gives both moderate candidates at least as much expected utility as in the two party equilibrium of stage two. This in turn depends on the ideological distance that separates the two moderate candidates.

Specifically, suppose that $\lambda > 1/4$. In this case, the two moderate candidates are so distant from each other that they cannot propose any policy in the interval $[t^2, t^3]$ that would be supported by voters in both moderate groups. Hence, the centrist party $\{2,3\}$ would lose the election with certainty, and it is easy to show that both moderate candidates would then prefer to move to stage two and reach the two party system described above.

Suppose instead that $1/4 \geq \lambda > 1/6$. Here, for a range of policies that depends on λ, the centrist coalition $\{2,3\}$ commands the support of moderate voters in both groups and if it is formed, it wins for sure. From the point of view of both moderate candidates, this outcome clearly dominates the two party outcome that would be reached in stage two, since they get higher expected rents and more policy moderation. Hence, the centrist party is formed for sure, and its policy platform depends on who is the agenda setter in the centrist party.

We summarize this discussion in the following:

Proposition 2 *If* $1/2 \geq \lambda > 1/4$, *then the unique equilibrium outcome under the single ballot is as described in Proposition 1. If* $1/4 \geq \lambda > 1/6$, *then the unique equilibrium outcome under the single ballot is a three party system with a centrist party,* $(\{1\}, \{2,3\}, \{4\})$. *The centrist party wins the election with certainty, and implements a policy platform that depends on the identity of the agenda setter inside the centrist party.*

We can then summarize the results of this section as follows. If the electorate is sufficiently polarized $(\lambda > \frac{1}{4})$, the single ballot electoral system penalizes the moderate candidates and voters. A centrist party cannot emerge, because the electorate is too polarized and would not support it. The moderate candidates and voters would prefer a situation where all candidates run alone, because this would maximize their possibility of victory and minimize the loss in case of a defeat. But this party structure cannot be supported, and in equilibrium we reach a two-party system where moderate and extremist candidates join forces. This in turn gives extremist candidates and voters a chance to influence policy outcomes[6]. If instead the electorate is not too polarized $1/4 \geq \lambda > 1/6$, then a single ballot system would induce the emergence of a centrist party. Extremist candidates and voters lose the elections, and moderate policies are implemented.

4 Two ballots

We now consider a closed two ballot electoral system. The two parties or candidates that gain more votes in the first voting round are admitted to the second ballot, that in turns determines who is elected to office. To preserve comparability with the single ballot rule, we start with exactly the same bargaining rules used in the previous section. Thus, all bargaining between candidates is done before the first ballot, under the same rules spelled out in section 2. In particular, candidates can merge into parties only before the first ballot. Once a party structure is determined, it cannot be changed in any direction in between the two ballots. We relax this assumption in the next subsection.

The features of the equilibrium under a two ballot system depend on other details of the model that were left unspecified in the previous sections. In particular, here we add the following two assumptions to those already made in section 2.

First, inside each extremist group, a constant fraction δ of voters is ideologically "attached" to a candidate. These attached individuals vote only if "their" candidate participates as a candidate on its own or as a member of a party. If their candidate does not stand for election (on its own, or as a member of another party), then they abstain. Note that this assumption plays

[6]Because moderate voters are in larger number than extremists and $\lambda \leq \frac{1}{2}$, this also means that the sum of total expected losses by citizens from equilibrium policies are larger when $\lambda > \frac{1}{4}$ and the centrist party cannot be formed, than when $\lambda \leq \frac{1}{4}$ and the centrist party can be formed.

no role in the single ballot system, since all candidates always participate in the election, either on their own or inside a party.

Second, we decompose the shock η to the participation rate of moderate voters in two separate shocks, each corresponding to one of the two ballots. Specifically, we assume that in the first ballot the size of group 2 voters is $\bar{\alpha} + \varepsilon_1$, while the size of group 3 voters is $\bar{\alpha} - \varepsilon_1$. In the second ballot, the size of group 2 voters is $\bar{\alpha} + \varepsilon_1 + \varepsilon_2$, while the size of group 3 voters is $\bar{\alpha} - \varepsilon_1 - \varepsilon_2$. The random variables ε_1 and ε_2 are independently and identically distributed, with a uniform distribution over the interval $[-e/2,\, e/2]$. This specification is entirely consistent with that assumed for η in the previous section. In fact, it is convenient to define here $\eta = \varepsilon_1 + \varepsilon_2$. Exploiting the properties of uniform distributions, we obtain that the random variable η now is distributed over the interval $[-e, e]$, it has zero mean and a symmetric cumulative distribution given by

$$\begin{aligned} G(z) &= \frac{1}{2} + \frac{z}{e} - \frac{z^2}{2e^2} \quad \text{for } e \geq \ z \geq 0 \\ G(z) &= \frac{1}{2} + \frac{z}{e} + \frac{z^2}{2e^2} \quad \text{for } -e \leq z \leq 0 \end{aligned} \tag{2}$$

Note that this specification implies that the first ballot reveals some relevant information about the relative chances of victory of one or the other moderate parties in the second ballot. This point is further discussed in the next subsection but plays no role here, since all bargaining is done before any voting has taken place.

Finally, we retain assumptions ($A1$) and (A2) in section 2. Clearly, these assumptions play an important role, because they determine who wins admission to the second round. In particular, assumption (A1) implies that a moderate candidate running alone always makes it to the second ballot, irrespective of whether the other moderate party has or has not merged with his extremist neighbor.

This does not mean that moderate parties always prefer to run alone, however. The reason is that, as spelled out above, a fraction δ of extremist voters is "attached" and will abstain in the second ballot if their candidate is not running. Merging with extremist thus presents a trade-off for the moderate candidates: a merger increases their chances of final victory, because it draws the support of these attached voters; but if they win, they get less rents and possibly worse policies. In the single ballot system, moderates faced a similar trade-off. But it was much steeper, because the probability of victory increased by 1/2 as a result of merging. With two ballots, instead, the change in the probability of victory is less drastic, and moderate parties may indeed choose to run alone. Whether or not this happens depends on parameter values, and on the expectations about what the other moderate party does.

Again, to solve for the equilibrium we have to work backwards. Thus, consider the second stage of bargaining. Suppose that candidates 3 and 4 have merged, while candidate 2 runs alone. Given the behavior of the attached extremists in group 1, candidate 2 will win in the second ballot if:

$$(1-\delta)\underline{\alpha}+\overline{\alpha}+\varepsilon_1+\varepsilon_2>\underline{\alpha}+\overline{\alpha}-\varepsilon_1-\varepsilon_2 \tag{3}$$

or more succinctly if:

$$\varepsilon_1+\varepsilon_2>\delta\underline{\alpha}/2$$

Given our assumption above on the support of the random variable η, if $e<\delta\underline{\alpha}/2$ then candidate 2 has no chance to win the elections when he runs alone. In this case, the double ballot does not offer any advantage to the moderate candidates, and the equilibrium is identical to the single ballot. Intuitively, if the share of their faithful voters is larger than any possible realization of the electoral shock, the extremist candidates retain all their bargaining power and the electoral system does not make any difference. Throughout this section we thus assume:

$$e\geq\delta\underline{\alpha}/2 \tag{4}$$

In this case, before any voting has taken place, the probability that candidate 2 wins given that 3 and 4 have merged is:

$$1-\Pr(\eta\leq\delta\underline{\alpha}/2)=1-G(\delta\underline{\alpha}/2)=1/2-h$$

where (4) implies:

$$\frac{1}{2}\geq h\equiv\frac{\delta\underline{\alpha}}{2e}(1-\frac{\delta\underline{\alpha}}{4e})>0$$

In the symmetric case in which no new party is formed and all four candidates run alone, the two moderate candidates win with probability 1/2 each. And in the other symmetric case of a two party system, each moderate-extremist coalition wins again with probability 1/2. Thus, the parameter h measures the handicap of running alone in a two ballot system, given that the opponents have merged. Using (A1), (A2) and (A3), it is easy to see that this handicap increases with the fraction of attached voters, δ, and the size of extremist groups, $\underline{\alpha}$, while it decreases with the range of electoral uncertainty, e.

The appendix proves that, if stage two of bargaining is reached, then the features of the equilibrium depend on whether the handicap of running alone, h, is above or below specific thresholds, $\bar{H} > \underline{H}$ and on the identity of the agenda setter inside the two prospective coalitions. More precisely:

Proposition 3 *Suppose that* $A(1)$, $A(2)$, $A(3)$ *hold and stage two of bargaining is reached. Then:*

(i) If $h < \underline{H} \equiv \frac{R}{4(2\sigma\lambda+R)}$ *the handicap of running alone is so small that both moderate candidates always prefer not to merge with the extremists. The unique equilibrium is a four-party system where all candidates run alone, and each moderate candidate wins with probability 1/2 with a policy platform that coincides with his bliss point.*

(ii) If $h > \bar{H} \equiv \frac{R}{4(2\sigma\lambda+R/2)}$, *the handicap of running alone is so large that both moderate candidates always prefer to merge with the extremists. The unique equilibrium is a two party system where moderates and extremists merge on both sides and each party wins with probability 1/2. If the moderate candidate is the agenda setter, then the policy platforms of each coalition coincide with the moderates' bliss points. If the extremist candidate is the agenda setter, then the policy platforms of each coalition lie in between the extremist and the moderate bliss points, and the distance between the equilibrium policy platforms and the moderates' bliss points is (weakly) decreasing in* h.

(iii) If $\underline{H} \leq h \leq \bar{H}$, *then two equilibria are possible. Depending on the players' expectations about what the other candidates are doing, both a two party or a four party system can emerge in equilibrium. In a two party system, the policy platforms are as described under point (ii).*

These results are very intuitive. If the handicap of running alone is very large, the two electoral systems do not make any difference, as moderates still always wish to merge with extremists, who then retain all their bargaining power. But if this handicap is small, then the bargaining power of the extremists is entirely wiped out, and the two ballot system induces that four party equilibrium which was unreachable under a single ballot rule because of the polarization of the electorate. In a sense, with the double ballot, voters are forced to converge to moderate platforms, by eliminating the extremist parties from the electoral arena. In intermediate cases, anything can happen, given candidates expectations on other agents' behavior. But notice that even in a two party system, the coalitions between moderates and extremists generally form on a more moderate policy platform compared to the single ballot case. The bargaining power of moderate candidates has increased, because a two-ballot system gives them the option of running alone without losing the elections with certainty, and this forces extremist candidates, when they are the agenda setters in the extremist-candidate coalitions, to offer moderate candidates a policy platform which is closer to their (the moderates) bliss points.

Next, consider stage one of the bargaining game. As before, one of the moderate candidates is randomly selected and makes a take-it-or-leave-it policy offer to the other moderate candidate. If the offer is rejected, the outcome described in Proposition 3 is reached.

As with a single ballot, the equilibrium depends on how much polarized is the electorate. If voters are very polarized (if $1/2 \geq \lambda > 1/4$), then there is no policy in the interval $[t^2, t^3]$ that would command the support of all moderate voters. Hence, the centrist party $\{2, 3\}$ would lose the election with certainty, and both moderate candidates would still prefer to move to the second stage of the bargaining game. Hence, if $1/2 \geq \lambda > 1/4$ the final equilibrium is as described in Proposition 3.

Suppose instead that $1/4 \geq \lambda > 1/6$. Here the centrist party would win for sure for a range of policy platforms. But this needs not imply that the centrist party is formed, because such a party would still have to reach a policy compromise and dilute rents among coalition members. If the handicap from running alone is sufficiently small (if $h < \underline{H}$), then both moderate candidates know that the four party system emerges out of the second stage game (see Proposition 3). Hence, by linearity of payoffs, they are exactly indifferent between forming the centrist party with a policy platform of $q = 1/2$ or running alone in a four party system. A slight degree of risk aversion would push them towards the centrist party, but an extra dilution of rents in a coalition government compared to the expected rents if they run alone would push them in the opposite direction. If instead the handicap from running alone is sufficiently large ($h > \bar{H}$), then the moderates are strictly better off with the centrist party, since the continuation game would lead them to merge with the extremists. Finally, for intermediate values of the handicap (if $\underline{H} \leq h \leq \bar{H}$), both outcomes are possible, depending on players beliefs about the continuation equilibrium.

We summarize this discussion in the following:

Proposition 4 *Suppose that* $A(1)$, $A(2)$, $A(3)$ *hold.*

(i) If $1/2 \geq \lambda > 1/4$, *then the unique equilibrium outcome under two ballots is as described in Proposition 3.*

(ii) If $1/4 \geq \lambda > 1/6$ *and* $h > \bar{H}$, *then the unique equilibrium outcome under two ballots is a three party system with a centrist party,* $(\{1\}, \{2,3\}, \{4\})$. *The centrist party wins the election with certainty, and implements the policy platform* $q = 1/2$.

(iii) If $1/4 \geq \lambda > 1/6$ *and* $h \leq \bar{H}$, *then two equilibrium outcomes are possible under two ballots: either the three party system with a centrist party described above, or the four party system described in part (i) of Proposition 3*

4.1 Two ballot with renegotiation

So far we assumed that no renegotiation is possible amongst the candidates in between the two rounds of voting. In practice, however, some renegotiation is likely to occur; this is indeed what happens in the Italian municipal elections, where candidates who did not pass the first voting round are allowed to endorse one of the contending opponents in the second round. As anticipated in the Introduction, endorsement after the first ballot is strictly regulated in Italy. First, endorsement need the written approval of both candidates, the candidate who passed the post and the first ballot defeated candidates wishing to endorse her. Second, the first ballot defeated candidates that endorse one of the two second round opponents are required to sign the latter original political platform, which cannot be modified between the two ballots. Third, after the formal endorsement is offered and accepted, the endorsing candidates participate in the legislative coalition together with the candidate and, in case of a victory, enjoy the majoritarian prize in the distribution of seats in the council (see below).

These institutional details suggest the following modelling strategy. We maintain the assumption that the policy cannot be renegotiated in between the two rounds. This is in line with the interpretation that the policy is dictated by the identity (ideology) of the candidate for major, which of course cannot be changed after the first round, and with the institutional Italian detail that policy platforms are presented before the first round of voting and signed by the endorsing parties[7]. Second, we assume that as a result of endorsing, the member of the winning coalitions share the rents from being in power; in line with the previous sections, we simply assume that rents are divided in half.

In our context, the consequence of an endorsement is to mobilize the support of the fraction δ of attached voters in the extremists groups. Under our assumption, these attached voters vote for the neighboring moderate candidate in the second round only if there is an explicit endorsement by the extremist politician. Otherwise they abstain.

Clearly, an excluded extremist politician is always eager to endorse, under our assumptions: by endorsing he has nothing to lose, but he can gain a share of rents in the event of an electoral victory. Furthermore, by endorsing, the extremist makes it more likely that the closer moderate candidate will win the elections, increasing the likelihood of an implemented policy closer to his bliss point[8]. The issue is whether moderate candidates seek an endorsement. They face a trade-off: an endorsement brings in the votes of the attached extremists, but cuts rents in half.

To describe the equilibrium, we work backwards, from a situation in which the two moderate candidates have passed the first ballot (endorsements can only arise if moderates have not already merged with extremists). We then ask what this implies for merger decisions before the first ballot takes place.

Suppose that both 2 and 3 have been endorsed by their extremist neighbors. By our previous assumptions, candidate 2 wins if $\varepsilon_1 + \varepsilon_2 > 0$. When decisions over endorsements are made, the realization of ε_1 is known, but ε_2 is not. Hence the probability that candidate 2 wins is

$$\Pr(\varepsilon_2 > -\varepsilon_1) = \frac{1}{2} + \frac{\varepsilon_1}{e} \tag{5}$$

[7] This is a bit far fetched. If the endorsed candidate wins, the endorsing extremist party is likely to obtain something in exchange, beyond the sharing of the majoritarian prize in the council. Typically, some members in the ruling cabinet, which then of course are going to affect policy. But given the Italian rules, it is still reasonable to assume that it is more difficult to change equilibrium policy if coalitions are formed after the first round of voting , than if coalitions are formed before this round.

[8] In a more general dynamic setting with asymmetric information, one should also consider that an extremist candidate may decide not to search an endorsement after the first ballot in order to signal his force to the moderate candidate, making the latter loses the elections and thus forcing him to accept a better deal before the first ballot in a future election (along the lines of Castanheida, 2004). This cannot happen in our model here because we assume that both $\underline{\alpha}$ and δ are determinist and known parameters.

where the right hand side follows from the assumption that ε_2 has a uniform distribution over $[-e/2, e/2]$. Notice that (5) also describes the probability that candidate 2 wins if neither candidate is endorsed, as in this case, by symmetry, both moderate candidates lose the same number of attached extremist voters.

Suppose instead that 3 has been endorsed by 4 while 2 did not seek the endorsement of 1. Now 2 loses the support of $\delta\underline{\alpha}$ voters, the attached extremists in group 1, while 3 carries all voters in group 4. Hence, repeating the analysis in (3), the probability that 2 wins is:

$$\Pr(\varepsilon_2 > \frac{\delta\underline{\alpha}}{2} - \varepsilon_1) = \frac{1}{2} + \frac{\varepsilon_1}{e} - \frac{\delta\underline{\alpha}}{2e} \tag{6}$$

if $\varepsilon_1 \geq \frac{\delta\underline{\alpha}}{2} - \frac{e}{2}$, and it is 0 if $\varepsilon_1 < \frac{\delta\underline{\alpha}}{2} - \frac{e}{2}$. Conversely, if 2 has been endorsed while 3 has not, then the probability that 2 wins is:

$$\Pr(\varepsilon_2 > -\frac{\delta\underline{\alpha}}{2} - \varepsilon_1) = \frac{1}{2} + \frac{\varepsilon_1}{e} + \frac{\delta\underline{\alpha}}{2e} \tag{7}$$

if $\varepsilon_1 \leq \frac{e}{2} - \frac{\delta\underline{\alpha}}{2}$ and it is 1 if $\varepsilon_1 > \frac{e}{2} - \frac{\delta\underline{\alpha}}{2}$.[9]

Hence, an endorsement by an extremist candidate increases the moderate's probability of victory by an amount proportional to the size of attached voters, $\delta\underline{\alpha}$. This gain in expected utility is offset by the dilution of rents associated with having to share power with the extremist candidate. It turns out that whether the gain probability is worth the dilution of rents or not depends on the realization of ε_1 relative to the following threshold:

$$\breve{\varepsilon} \equiv \frac{\delta\underline{\alpha}}{2}(1 + \frac{4\sigma\lambda}{R}) - \frac{e}{2}$$

where $\breve{\varepsilon} \lesseqgtr 0$. If $\varepsilon_1 < \breve{\varepsilon}$, the probability of victory for 2 is so low that he always prefers to be endorsed by the extremist no matter what 3 does. Conversely, if $\varepsilon_1 > \breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2}$, then the probability of victory for 2 is so high that he prefers no endorsement no matter what candidate 3 does. In between (if $\breve{\varepsilon} \leq \varepsilon_1 \leq \breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2}$), then candidate 2 prefers to seek the endorsement of the extremist if 3 has also been endorsed, while 2 prefers no endorsement if 3 has not been endorsed. Candidate 3 behaves symmetrically, except that for him the relevant issue is whether $-\varepsilon_1$ is below or above these same thresholds.

[9] By (4), $\Pr(\varepsilon_2 > \frac{\delta\underline{\alpha}}{2} - \varepsilon_1) < 1$ $\Pr(\varepsilon_2 > -\frac{\delta\underline{\alpha}}{2} - \varepsilon_1) > 0$ and for any $\varepsilon_1 \in [-e/2, e/2]$.

The equilibrium then depends on whether these thresholds are positive or negative. Specifically, the appendix proves that, once candidates 2 and 3 compete on the second round of elections, the equilibrium is as follows (throughout we assume that (A1-A3) hold):

Proposition 5 *(i) Suppose that* $\breve{\varepsilon} > 0$. *Then the equilibrium is unique and at least one of the two moderate candidates always seeks the endorsement of his extremist neighbor. If* $\varepsilon_1 \in [-\breve{\varepsilon} - \frac{\delta\underline{\alpha}}{2}, \breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2}]$ *then both candidates seek the endorsement of their extremist neighbor. If* $\varepsilon_1 > \breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2}$ *then 3 seeks the endorsement while 2 does not. If* $\varepsilon_1 < -\breve{\varepsilon} - \frac{\delta\underline{\alpha}}{2}$ *then 2 seeks the endorsement while 3 does not.*

(ii) Suppose that $\breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2} < 0$. *Then the equilibrium is again unique and at most one of the two moderate candidates seeks an endorsement by his extremist neighbor. If* $\varepsilon_1 \in [\breve{\varepsilon}, -\breve{\varepsilon}]$ *then no moderate candidate seeks the endorsement of the extremist. If* $\varepsilon_1 > -\breve{\varepsilon}$, *then 3 seeks the endorsement of 4 while 2 seeks no endorsement. If* $\varepsilon_1 < \breve{\varepsilon}$, *then 2 seeks the endorsement of 1 while 3 seeks no endorsement.*

(iii) Suppose that $\breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2} > 0 > \breve{\varepsilon}$. *If* $\varepsilon_1 \in [-\breve{\varepsilon}, \breve{\varepsilon}]$, *then multiple equilibria are possible: either both moderate candidates seek an endorsement by their extremist neighbor or none of them does. For all other realizations of* ε_1 *the equilibrium is unique. If* $\varepsilon_1 \in (-\breve{\varepsilon}, \breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2}]$ *or if* $\varepsilon_1 \in (\breve{\varepsilon}, -\breve{\varepsilon} - \frac{\delta\underline{\alpha}}{2}]$ *then both moderate candidates always seek the endorsement of the extremist. If* $\varepsilon_1 > \breve{\varepsilon} + \frac{\delta\underline{\alpha}}{2}$ *then 3 seeks the endorsement of 4 while 2 does not seek any endorsement; and symmetrically, if* $\varepsilon_1 < -\breve{\varepsilon} - \frac{\delta\underline{\alpha}}{2}$ *then 2 seeks the endorsement of 1 while 3 does not seek any endorsement.*

These results are very intuitive (see also figures 2 to 4 in the Appendix for an illustration of the results). What they say is that endorsement depends on the realization of ε_1 with respect to a given threshold $\breve{\varepsilon}$; and that irrespective of the realization of ε_1, endorsement by at least one of the candidate is the more likely the larger is this threshold. Notice that $\breve{\varepsilon}$ is increasing in $\delta, \underline{\alpha}$, σ, λ and decreasing in R and e. Thus, merging with the extremist is the more likely after the first ballot, the more important the extremists are in determining the final results (that is, the larger is the size of the attached extremist voters) and the more important is the ideological component in the moderate candidate utility function and the dispersion in the electorate (because this increases the utility loss of having the other candidate wins the elections, if no endorsement with the extremist takes place). Conversely, the threshold is lower and endorsements are less likely, the larger are the rents (because then the larger are the costs of cutting them in half to endorse with the extremist) and the larger is the uncertainty in voters' turn out (because this makes less likely that extremists are determinant in the final result).

Next, knowing that this is the outcome if both moderate candidates reach the second round of elections, consider what happens before the first round. Again, start backwards, and suppose that the moderate candidates bargain with the extremists over party formation. Now, the moderates lose any incentive to merge with the extremists before the first round of elections. They know that they will always make it to the second round[10]. They also know that, after the first round, they will always be able to get the endorsement of the extremists if they wish to do so, since the extremists are eager to share the rents from office. But waiting until after the first round gives the moderates an additional option: if the shock ε_1 is sufficiently favorable, then they can run alone in the second round as well, without having to share the rents from office. This option of waiting has no costs, since the extremists are always willing to endorse. Hence the option of waiting and running alone in the first round of elections is always preferred by the moderate candidates to the alternative of merging with the extremists. We summarize this discussion in the following:

Proposition 6 *Suppose that stage two of bargaining is reached. Then the unique equilibrium outcome at the first electoral ballot is a four party system where all candidates run alone and each moderate candidate passes the first post with probability 1/2 on a policy platform that coincides with his bliss point. After the first round of elections, endorsements by the extremists take place on the basis of the realization of the shock* ε_1 *as described in Proposition 5.*

Finally, in light of this result, consider the first stage of the bargaining game, where the two moderate candidates bargain with each other over the formation of a centrist party. If $\lambda > 1/4$, then as in Propositions 2 and 4 the electorate is too polarized to sustain the emergence of a centrist party and bargaining moves to stage 2 (and then to the four candidates running alone at the first electoral ballot). If instead $1/6 < \lambda \leq 1/4$, then the centrist party is feasible. By forming the centrist party the two moderate candidates win

[10] This is of course a consequence of A.2. We discuss briefly what happens when this assumption is relaxed in section V.

with certainty but have to share the rents in half and achieve some policy convergence. By giving up on this opportunity, the two moderate candidates know that they would end up in the equilibrium outcome described in Proposition 5. Here, each moderate candidate passes the post with probability $1/2$ on his preferred policy platform; but his expected share of rents is now strictly less than $R/2$, since with some positive probability the moderate party is forced to seek the endorsement of the extremist and this dilutes his expected rents (or alternatively, even the first ballot shock is so favorable that the moderate party will prefer to run alone his expected probability to win the final elections is less than $1/2$ since the other moderate party will accept the endorsement of his extremist). Hence, forming the centrist party always strictly dominates the alternative of running separately at the first round of elections. The centrist party is formed with certainty on a policy platform that is tilted towards the bliss point of whoever happens to be the agenda setter in the centrist coalition (since there are positive expected gains from forming the centrist party, these gains accrue to whoever happens to be the agenda setter in the centrist party).

We summarize this discussion in the following:

Proposition 7 *(i) If* $1/2 \geq \lambda > 1/4$, *then the unique equilibrium outcome under two ballots is as described in Proposition 5.*

(ii) If $1/4 \geq \lambda > 1/6$, *then the unique equilibrium outcome under two ballots is a three party system with a centrist party* $(\{1\}, \{2,3\}, \{4\})$. *The centrist party wins the election with certainty, and implements a policy platform that depends on the identity of the agenda setter inside the centrist party.*

Summing up, our model offers clear cut predictions concerning both coalition formation and policy determination under the two electoral rules. Beginning with the former, our model predicts the following. When the electorate is not very polarized $(1/4 \geq \lambda)$ a centrist party will always form under both electoral systems and regardless whether renegotiation after the first ballot is or is not allowed. Only in the case where renegotiation is not allowed (e.g. ex post bargaining is very costly) and the handicap of running alone is very small (i.e. $\delta\underline{\alpha}$ is very small), the electoral system may make a difference, in the sense that the two moderate candidates, even if they could form a winning centrist party at the first round of the two ballots election system, may nevertheless decide to run alone, as there are however sure to be one of the two winning candidates and could then implement their preferred policy without sharing rents (see Proposition). Renegotiation destroys this possibility, because moderates then know that with some positive probability they will seek the endorsement of the first ballot losing extremists (Proposition 6). The electoral system makes instead a huge difference when the electorate is polarized ($\lambda > 1/4$) so that a winning centrist coalition cannot be sustained. In this case, under a single ballot rule, two large extremist-moderate coalitions will form for sure. Viceversa, under a two ballot system, depending on the size of the handicap of running alone and the possibility of renegotiation, a larger set of equilibria in coalition formation is possible. If renegotiation after the first ballot is prohibited or very costly, there are equilibria where all parties run alone at the first round, as there are equilibria where the same moderate-extremists coalitions form at the first ballot (Proposition). On the other hand, if renegotiation between the two ballots is allowed, we should never observe the formation of coalitions at the first round of the two ballot system, as moderates have always the possibility of being endorsed by extremists after the first ballot (Proposition).

But our results are mostly striking with respect to policy determination. They imply that the two ballot electoral system is an effective way to reduce the bargaining power of extremists and to induce therefore more moderation in policy. Even if an extremist-moderate coalition forms in the first round of the two ballot system, it usually does so on a more moderate policy platform than under the single ballot system (Proposition). And in many cases, as we saw above, this coalition simply does not form under the two ballot system, implying that one of the moderate candidate wins for sure and implement his preferred policy. Under our assumptions on the bargaining process, this is true even if renegotiation and endorsement is allowed after the first round of the two ballot system.

Our next task is then to take these predictions to data. But before doing so, it is worth questioning further the robustness of our results, by considering different sets of assumption on the parameters of our problem.

5 Extensions

5.1 Low polarization of the electorate

To start with, consider first the case where the electorate is not very polarized, so that $\lambda < 1/6$[11]. This means that the distance on the policy line between the bliss point of the extremists and that of the closer moderate is larger than the distance between the bliss points of the two moderates. In turn, this complicates the analysis of the model, because it implies that at stage 2 of our bargaining game, when the extremists are the agenda setters, they do not have any longer a dominant strategy. They cannot propose their bliss points to the moderates (because otherwise all voters of the closer moderate group would now prefer to vote for the other moderate, if this runs alone), and in general the willingness of a moderate to accept a proposal made by his closer extremist now also depends on the proposal made by the other extremist, as well as on the expectations held by the first moderate on the fact that the other estremist's proposal will be accepted or not by the other moderate. In order to solve the model, it is then necessary to solve first explicitly the subgame between moderates which follows the two extremists's simultaneous policy proposals, and then move backwards to the first stage of the game to pinpoint the equilibrium policy proposals by the two extremists. The Appendix provides details. It turns out that the subgame among moderates admits multiple equilibria in both pure and mixed strategies, which in turn implies that there are also multiple subgame perfect equilibria in the complete bargaining game. The next proposition summarizes our conclusions:

Proposition A.1 *Assume $\lambda < \frac{1}{6}$ and suppose that under the single ballot electoral rule stage 2 of the bargaining game is reached. Then (i) if the moderates are the agenda setters, they just propose their bliss points to their respective extremist. These proposals are accepted and parties (1,2) and (3,4) form on the policy platforms of the moderates. If instead the extremists are the agenda setters, then there are three possible subgame perfect equilibria of the game, depending on extremists' expectations about moderates' equilibrium behavior in the subsequent subgame. (ii) At one such equilibrium, candidate 1 proposes to candidate 2, $q^{12} = \frac{1}{2} - 3\lambda$, and candidate 4 proposes to candidate 3, $q^{34} = \frac{1}{2} + 3\lambda$. Both proposals are accepted and parties (1,2) and (3,4) form on the proposed policy platforms. (ii) At the other two equilibria, either candidate 4 proposes $q^{34} = \frac{1}{2} + 3\lambda$ and 1 proposes $q^{12} = \frac{1}{2} - 2\lambda$; or candidate 4 proposes $q^{34} = \frac{1}{2} + 2\lambda$ and candidate 1 proposes $q^{12} = \frac{1}{2} - 3\lambda$. In both cases all proposals are accepted and parties (1,2) and (3,4) form on the proposed*

[11] In the limiting case $\lambda = \frac{1}{6}$ both the equilibria discussed in the previous sections and the one discussed here are possible.

policy platforms.

Thus, a reduction in the polarization of the electorate below the threshold $\lambda = \frac{1}{6}$ forces the extremists, when they are the agenda setters, to moderate their policy proposals, moving them closer to the moderates' bliss points. Notice for example that $\lambda < \frac{1}{6}$ entails $\frac{1}{2} - 2\lambda > \frac{1}{2} - 3\lambda > 0$, so that at no equilibrium of the bargaining game candidate 1 can now propose his bliss points to candidate 2 (similarly for candidate 4). But notice also that $\lambda < \frac{1}{6}$ implies $\lambda < \frac{1}{4}$, so that if moderates joined forces at the first stage of the bargaining game they would win the elections for sure, agreeing further on a policy platform which is better for them (in expected terms) that any of the possible equilibria resulting from stage two of the bargaining game. This implies that our proposition 2 still holds, and with $\lambda < \frac{1}{6}$ the unique equilibrium outcome under the single ballot is a three party system with a centrist party, $(\{1\}, \{2,3\}, \{4\})$.

Repeating the argument, it is easy to conjecture that the same conclusions should follow even under the two ballot electoral rule. Under this rule, when they are the agenda setters at stage 2 of the bargaining game, the extremists are constrained in their policy proposals by both the fact that $\lambda < \frac{1}{6}$ and by the fact that with some probability moderates can win the elections even if they run alone. This should lead them to moderate their proposals even further, in order for them to be accepted by the moderates. But again, as $\lambda < \frac{1}{6}$ implies $\lambda < \frac{1}{4}$, a winning centrist party is feasible and moderates would still prefer to form a coalition among them rather than with the extremists as this guarantees them higher rents and better policies. The conclusion is that all our previous statements should be unaffected by letting polarization falls below the threshold $\lambda = \frac{1}{6}$,[12] although the analytical details become more complicated.

[12] Notice from Proposition A1 that in the limiting case $\lambda \to 0$, all policy proposals in the second bargaining stage collapse to $q = \frac{1}{2}$; extremists could not propose anything different if they want their proposal to be accepted by the moderates. This means that the moderates would get their preferred policy either if they join in a coalition between them or if they form two separate moderate-extremist parties. But again moderates would still prefer to form a single moderate party, as in this case they would earn $\frac{R}{2}$ for sure, while they would only earn $\frac{R}{4}$ expected rents if they joined the extremists.

5.2 Moderates as the smaller parties

As a second extension, consider then the case where the moderates are the smaller parties and the extremists the larger ones; would our results still go through with inverted roles, implying that the two ballot electoral system really supports whoever is the larger party, rather than moderate policy in itself? In many cases, the answer is yes, but with an important caveat to keep in mind. In general, moderate parties have an advantage with respect to extremists; they can find an agreement between them, while it is hard for parties at the opposite extremes of the political spectrum to strike a convincing deal. Extremist voters would not believe in, and possibly not vote for, a coalition formed by opposite extremist candidates. In our model, we capture this real world asymmetry by assuming that moderates move first and may strike a deal with both the other moderate and the extremist, while extremists move later and are forced to seek an alliance with the closer moderate only. But this asymmetry implies that the two ballot system may still advantage the moderates even though they are a minority, in the specific sense of making the formation of a moderate centrist party possible, when this was not possible under the single ballot rule. The basic reason is that under the two ballot system –differently from the single ballot case– what matters is not to win the first round, but to pass the first electoral test and win the final elections. And a moderate party that manages to pass the fist post has a larger probability to win the final elections, as it can then collect the votes of (the not-attached) voters of the extremist party that is no longer running at the final ballot.

To illustrate the point, consider the following example. Suppose now that moderates have size $\underline{\alpha}$ and extremists size $\overline{\alpha}$, while, for simplicity, everything else is kept unchanged. So, moderates first bargain among them and then (possibly) with the extremists, according to the rules described in section 2; and we still assume that groups with size $\overline{\alpha}$ (now the extremists) are subjected to a symmetric electoral shock, with the properties described in the previous sections[13]. Thus, we retain both assumptions A.1 and A.2, and the decomposition of the shock between the two ballots introduced in section 3. But we also add here, for reasons which will become apparent shortly, the assumption that

[13]Of course, in this case, the shock must be interpreted as a shock to voters' participation, rather than a shock which redistributes votes across moderates.

$$\frac{e}{2} > (\overline{\alpha} - 2\underline{\alpha}) > 0 \qquad A.4$$

In words, A.4 implies that the size of each group of extremist voters is larger than the sum of the two moderate ones, but that at each ballot, electoral uncertainty is large enough to modify this ranking for some realization of the shock. Let us also assume $1/4 \geq \lambda > 1/6$, so that a viable centrist party is feasible (there exists a policy proposal which would be supported by all moderate voters, if extremists run alone). Consider then again the two electoral rules in this context.

Under the single ballot rule, it is clear that moderate candidates will never form a centrist party at stage 1 and prefer to move to stage 2 instead. The reason is that under our assumptions on the distribution of the electoral shock and A.4, a centrist party, while viable, would always be defeated at the single ballot elections by (at least) one of the two extremists. Furthermore, the consequences of running alone would be disastrous for moderates even on policy grounds, as now the extremists' preferred policies would be implemented with probability $\frac{1}{2}$ each. On the other hand, if moderates decide to go on to stage 2, they now become essential players in the moderate-extremist coalitions, and it is easy to see that under our specified bargaining rules, the results in Proposition 1 would go through unchanged. Thus, a two party system with a coalition of extremists and moderates on each side will form, each one winning with probability $\frac{1}{2}$, moderates will then earn expected rents equal to $\frac{R}{4}$, and each of the policies preferred by the four candidates will be implemented with equal probability, a clear improvement with respect to the prospective of running alone for the moderates.

But consider now the two ballot electoral rule. Suppose a centrist party forms at stage 1 and runs at the first ballot election. Under our assumptions, the centrist party can still not possibly win at this ballot; but, if the negative electoral shock which hits one of the two extremists is large enough, it can pass the post and goes to the second turn. This will happen if the realization of ε_1 is such that $2\underline{a} > \overline{a} + \varepsilon_1$ (which, by symmetry, implies $\overline{a} - \varepsilon_1 > 2\underline{a}$, so that the extremist hit by the positive shock will win the first ballot). And under our assumptions on the distribution of ε_1 this occurs with probability $p_1 = 1 - \frac{2(\overline{a} - 2\underline{a})}{e}$, a positive number by assumption A.4. Thus, the centrist party will reach the second ballot with probability $p_1 > 0$;and given that it has reached the second turn, under our assumption on voters' behavior, it will then win the final ballot if

$$\overline{a}+\varepsilon_1+\varepsilon_2<2\underline{a}+(1-\delta)(\overline{a}-\varepsilon_1-\varepsilon_2)$$

Let p_2 be the probability of this to occur[14]; and notice that $p_2=0$, if $\delta\geq(2-\frac{\overline{a}}{\underline{a}+\frac{e}{4}})\equiv\overline{\delta}$ and $p_2=1$, if $\delta\leq\frac{2(\underline{a}-e)}{(\overline{a}-e)}\equiv\underline{\delta}$. Thus, for $\overline{\delta}>\delta>\underline{\delta}$, $1>p_2>0$. In words, and quite intuitively, if the share of the attached voters is not too large, the centrist party has some possibility of winning the final elections under the two ballot rule, while this was impossible, under A.4, under the single ballot rule. In turn, whether the moderates decide to form a centrist party at the first stage or move on to bargain with the extremists at the second stage, depends on their expected utility under the two different scenarios. Computing these expected utilities, it is easy to see that moderates will prefer to form a centrist party at stage 1 and run alone at the elections if $p_1p_2>\frac{1}{2}$; that is, if the joint probability of passing the first post and winning the final elections is larger than one half. Inspection of p_2 shows this is certainly a possibility; for instance, for $\delta\leq\underline{\delta}$, this will certainly occur if $\frac{e}{4}>(\overline{\alpha}-2\underline{\alpha})$, that is, if the moderate voters, when joining forces, are sufficiently close in size to each extremist party. Notice further that extremists are completely passive in this process; given our assumed bargaining rules, if moderates decide to form a centrist party, there is nothing the extremists can do, except running alone at the elections and risk a defeat, although they represent the majority of the electorate.

The example above is of course rather artificial, as we have assumed, for simplicity, that the distribution of the electoral shocks does not change even if one of the extremist is not running anymore at the second ballot. But it would not be difficult to construct other examples, changing the distribution of the electoral shocks, which still produced the same results. More generally, as long as our two key assumptions here (that moderates can form a coalition between them more easily than extremists can do, and the assumption that once reached the second ballot, some extremist voters will prefer to vote for the moderates rather than risking that the opposite extremist wins the elections) are taken to be realistic, the basic insight that the two ballot electoral system may help the moderates even if they are a minority seems to be robust.

[14] Given our assumptions on the distribution of the two shocks, p_2 could be explicitly computed, but that is not necessary given our objectives here.

5.3 Forming ex ante coalitions in a dual ballot system with renegotiation

A striking result of our previous analysis is that a moderate-extremist coalition should never form before the first round in a double ballot system, once renegotiation is allowed between the two rounds. The moderate candidate would refuse any ex-ante agreement, because he knows for sure that he can always reach a better deal with the extremist after the first round, if he wishes to do so. But this result is a consequence of our assumption A.1 which ensures moderates that they will always be the ones to pass the first turn if they run alone at the first ballot. Relaxing this assumption, there are cases where moderate candidates, if unsure to pass the post, may prefer to strike a deal with the extremists before hand, as a form of insurance against the risk of having to bargain, at worse terms, with the extremists after the first ballot. Interestingly, by the asymmetry among extremists and moderates, this turns out to be true even maintaining the linearity of payoffs of the players (i.e. risk neutrality), although introducing risk aversion would reinforce the argument even further. Details are available by the authors on request.

5.4 Strategic voters

We have assumed so far that voters vote sincerely; with the exception of the "attached" voters when their candidate is not running, in our model voters always vote and always cast a ballot in favour of the candidate or the party which proposes them a policy which is closest to their preferences. But this implies that we do not allow rational voters to react to the incentives to vote strategically which may be offered by the electoral system[15]. Furthermore, as these incentives may well depend on the characteristics of the electoral system itself, this raises the question on the robustness of our results. That is, how would our results change if we allowed voters to cast a strategic vote?

Unfortunately, it is not easy to answer this question, as there is no agreement in the literature on either how strategic voting should be modelled in

[15] By the Gibbard-Sattherwhite theorem, any social decision rule which is not dictatorial offers incentives to the agents to lie; that is, in our context, offers voters incentives to cast a strategic vote, rather than a sincere one.

our context or, for what matters, on its importance in reality. In general, the empirical literature suggests that strategic voting is a reality, but that its relevance is somewhat limited (see Cox, 1997 and the empirical literature quoted there). For instance, Degan and Merlo (2006), in a recent empirical study on the US elections for the House and the Senate in the period 1970-2000, find that only 3% of the observed individual voting profiles could not been made consistent with sincere voting, a figure well below measurement error. Sinclair (2005), in an empirical analysis on the reasons for the "failure" of Duverger's law for the UK, finds support for strategic variables in influencing voters' decision, but also finds this support to be very limited, suggesting that there is an expressive component in voting behavior which generally overcomes strategic considerations. Furthermore, while there is an extensive theoretical literature on strategic behavior in single ballot electoral systems under different electoral rules (e.g. Myerson and Weber, 1993; Fey, 1997), very little work has been devoted to analyze this behavior for dual ballot electoral systems[16].

In one such study, for example, Cox (1997) argues the following: 1) the incentives to play strategically in the first round of the dual ballot system should be similar to the incentives to play strategically in single ballot electoral systems[17]; but 2) that in practise, these strategic incentives should be lower than in the single ballot system. In particular, for the first point, Cox (1997) proves theoretically that a version of the "M+1" rule holds for double ballot electoral systems too; in a dual electoral system with M candidates passing to the final ballot, no more than "M+1" candidates should be considered as viable by rational voters, implying that all candidates that are perceived by voters as ranked below the M+1 position should lose all electoral support at the first ballot. But as Cox hastens to add " in practise strategic voting in the first round of runoff elections is probably much rarer than the theoretical benchmark established by the model. This is partly because the informational preconditions of rational expectations are greater, and partly because optimal strategies are more complex in dual-ballot than

[16] As well known, Duverger's own suggestion was the double ballot system would not produce incentives to play strategically in the first round of voting, a point criticized by Sartori.

[17] In the second round, with only two candidates running, there is of course no incentives for voters to vote strategically, at least as long as voters are only interested in the results of the present elections (and not for example, on the effects of the results of the present elections on future elections).

in a single-ballot system" (Cox, 1997: 124).

This clearly does not help us much in answering the question we raised above. Depending on how we decide to model strategic behavior in our setting, results could change considerably. But there is at least a useful benchmark which we can establish. Suppose that is common knowledge that only a fraction of extremist voters, δ, are "attached" to their candidates, meaning as above that they only vote if their candidate is running at the elections, either alone or in a coalition with a moderate[18]. Suppose further that all other extremist voters are now 'short term' rational, meaning that they vote for whoever offers them the higher expected utility from the current elections, that is, taking also into account the probability that a candidate has to win the elections when running. Consider again the single ballot system under these new behavioral assumptions. The incentives for the moderate candidate to form a coalition with the extremist now change considerably. The moderate knows that if he runs alone at the single ballot, he can now attract all the rational extremists; for the usual Duvergian argument, rather than voting for the extremist candidate, who offers them their preferred policy but who has not chance of winning if he runs alone, the not attached extremist voters would prefer to vote for the closer moderate, who offers them a next-to-preferred policy but who has some positive probability of winning if he runs alone. This would weaken the bargaining powers of the extremist candidates, forcing them to moderate their requests on the policy platform when contracting with the moderates at the second stage of the bargaining game. Indeed, extending the argument, it is easy to see that under the assumptions of section 3, the effect of introducing this kind of strategic behavior in our model would be to erase any difference between the two electoral systems; the same coalitions would form and on the same policy platform under both the single ballot system and the dual ballot systems, as the bargaining powers of the two candidates, extremist and moderates, are now exactly the same under the two systems. In other words, contrary to all our previous results, the electoral system would not matter if we allowed some voters to vote strategically, in the specific sense explained above. This suggests that our results are extremely sensitive to the assumption of sincere

[18]Note that in the present context the attached voters could be the voters for whom the expressive component of voting (the utility they receive for voting for "their" candidate) overcomes the strategic component (the possibility of affecting the final results). Or these voters could be more sophisticated voters, who take into account the effect of abstaining today on future elections.

voting; but it also suggests an easy way to check the relevance of strategic voting in our context. If it turns out empirically that the electoral system matters, for both coalitions formation and policy determination, in the direction suggested by our previous model, this implicitly brings support to our behavioral assumptions. Strategic voting may possibly play a role, but not enough to change qualitatively the results that we obtain in a simple model with sincere voting.

6 Data set and Italian institutions

6.1 Italian municipalities' electoral rules

Let us start by illustrating in more details the characteristic of the institutional example we are going to study. Before 1993, elections at the municipal level in Italy took place according to a simple pure proportional parliamentary system for any population size in our sample. Voters voted for parties to elect the municipal legislature (Council); the Council then elected the Mayor and the municipal executive (Giunta); and the Council could dismiss at any time the Giunta and the Mayor, and form another executive and elect a new Mayor. After 1993, rules were changed, moving to a still proportional electoral system but with the direct election of the Mayor, a "majority prize" and different rules for municipalities below and above 15,000 inhabitants.

Below the threshold, parties form ex ante coalitions which support the candidates who run for mayor. The list includes, in addition to the name of the candidate for Mayor, the list of the candidates for the municipality's Council. Voters cast a *single vote* for the Mayor and the Coalition; they can express a preference for the candidates to the Council, so reverting or changing the list presented by the coalition. The candidate who gets more votes, becomes Mayor and the coalition which supports the winning candidate earns a *majority prize;* 2/3 of all seats in the Council are assigned to the coalition supporting the winner, with seats shared across parties in the winning coalition on the base of the order on the list and the preferences expressed by voters. Losing coalitions share the remaining 1/3 of the seats on the base of the votes collected by their candidates. Once elected, the Mayor forms the Giunta (executive), which may or may not include members of the Council, and can at any time dismiss one or more members of the Giunta and substitute them with someone else. As already stressed, the Mayor needs a compliance vote by the Council to rule, but if the Mayor is dismissed by the Council or if the Mayor resigns, new elections need to take place.

Above the threshold, the electoral rules are more complex. Political parties form explicit ex ante coalitions to support a candidate for Mayor, but there is no longer a single list supporting a single candidate. Several lists (parties or coalition of parties) may support the same candidate. Also, voters may also cast a *disjoint vote.* Voter may cast a vote for one of the party which supports a candidate; in which case, the vote immediately counts also as a vote for the candidate supported by that specific party. Or he might vote for the candidate only; in which case, the ballot counts for the Mayor but not for the supporting parties. Finally he might actually vote for a candidate and *for a party which supports a different candidate*; in which case both votes count, one for electing the Mayor and one for electing the Council. Again, voters can express a single preference for a candidate to the Council, by writing her/his name on the electoral paper. Rules for electing the Mayor are as follows:

-If a candidate gets more than 50% of votes at the first round, s-he becomes immediately Mayor.

-If no candidate gets more than 50% of votes at the first round, the two most voted candidates run again at the second ballot. At the second ballot, voters only vote on the two candidates for Mayor, not for parties.

-Between the two rounds of voting, new coalitions can be formed ("accorpamento" is the Italian word for this explicit ex post coalition). If one of the first round losing candidate wishes to endorse one of the two second round runner and this endorsement is accepted by the second round runner and this candidate wins, the parties who supported the losing candidate also share the mayority prize (on the basis of the first round electoral result), provided that this prize is given. But as already explained to form a coalition after the first ballot: 1) both the endorsing and the endorsed candidate need to agree; 2) the first round losing candidate needs to sign the policy platform proposed before the first turn by the second round runner;

-At the second round, the candidate that receives more votes wins and becomes Mayor.

Once elected, the Mayor forms the Giunta and rules the Municipality according to the rules explained above for the municipalities below the threshold.

The rules for assigning seats in the Council are as follows.

-If a Mayor is elected at the first round, and the supporting lists also get more than 50% of votes, there is then a "majority prize". The winning lists get at least 60% of the seats in the Council and the losing ones 40%. Seats are attributed among parties on the bases of the first round result. If the original share of votes of the winning lists was larger than 60%, they retain this additional advantage and seats are distributed across parties according to a simple proportional rule (d'Hondt method). By the same token, if the Mayor is elected at the second round and the no other coalitions of parties supporting a *different* candidate has obtained more than 50% of votes at the first round, the lists supporting the winning candidate (included the ones added in the between ballots bargaining period) get the "majority prize".

-If a Mayor is elected at the first round, but the supporting lists get less than 50% of votes, there is no "majority prize" and seats in the Council are distributed across parties according to the proportional rule (d'Hondt method). By the same token, if the Mayor is elected at the second round but another coalitions of parties supporting a *different* candidate has obtained more than 50% of votes at the first round, there is no "majority prize" and seats in the Council are again distributed across parties according to the proportional rule (d'Hondt method).

The complications of this last case derive of course by the possibility offered to voters to cast a *disjoint vote*. Since voters can simultaneously vote for a candidate and for a party supporting a different candidate, it might be possible that a Mayor is elected without a majority or even with an opposing majority in the Council, in which case, of course, the majority prize is not assigned. Fortunately, these cases are rare and in general not viable (recall that the Council can always dismiss the Mayor, providing it is willing to risk a new elections, and the Mayor can always resign, so again triggering a new election). However, it is worth recalling, to interpret the results to follow, that above the threshold, first round votes for a candidate to Mayor, and votes for the supporting coalitions may not, and in general do not, coincide.

6.2 Extremists and moderates

Our theoretical analysis is based on the distinction between "moderate" parties and voters and "extremist" ones. In order to perform our analysis we need to identify which these parties are in the Italian context. At the national level, this is a relatively easy task. The Italian political landscape is neatly divided in five groups of parties. At the right of the political spectrum, Alleanza Nazionale (AN) and Forza Italia have always joined forces since the foundation of the latter party in 1994, and are by now so close that there is talk of joining in a single party. They are the bulk of the Center-Right coalition, and so we assign them this label. Together, they count for 30-35% of votes at the national level. On the left side, DS (socialdemocrats) and Margherita (mostly, social catholics) have always been allied and have recently joined in a single party, the Democrats. We define them the Center-Left coalition. These parties count for around 30% of votes at the national level. There is then a handful of small centrist parties, the heirs of the Christian Democrats and Socialists. They have sometimes changed alliances between right and left and we then assign them to the Center Right and Center Left according to their prevailing alliance at the national level. At the left of the DS, there is a vast area (around 6- 10% of votes at national level) of small parties, belonging to the so called "radical left", whose principal party is Rifondazione Comunista (comunist party), around 5-6% of votes at the national level. On the other side of the spectrum, there are no significant parties at the right of AN. But there is a strong regional party, the Lega North, which while pretty small at the national level (around 4-5% of votes), is very strong in some zones of the Northern Regions –Veneto, Piemonte and mainly Lombardia–where is often the largest party.

For our tasks, it is relatively easy to identify the Center-Right and the Center–Left as the "moderate" parties and Rifondazione and Lega as the "extremists" ones. With all their differences, Center-Right and Center-Right share some fundamental values, a support for market economies and fundamental alliances at the international levels, in particular with Nato and the European Union. On the contrary, both Rifondazione and Lega are anti-market and anti-Europe parties. On the other hand, Rifondazione and Lega differ radically on some basic issues such as immigration policy, tax policy, and, of course, fiscal federalism and redistribution. On some aspects, such as immigration policy, Lega is closer to some of the extreme racist party which abound in Europe.

Putting it differently, the policy agenda of both Lega and Rifondazione is radically different from the policy agenda of the majority of the Parliament. Further, while occasionally allied to one of the two main coalitions, they have always marked their distance to the latter, occasionally contributing to their defeat. In 1995, the Center-Right government fell when Lega decided to withdraw its support, and the decision by Lega to run alone at the ensuing 1996 national political elections was instrumental for the defeat of the Center-Right coalition. By the same token, the decision of Rifondazione to renege its alliance with the Center-Left in 1998, was decisive in provoking the fall of the first Prodi's government and the subsequent defeat of the Center-Left coalition at the political elections of 2001. Thus, in spite of their relative limited share of votes at the national level, both parties have shown to be essential players for the electoral success of the respective coalitions, and so when in powers have managed to affect coalition policies far beyond their share of votes. Table X, in the Appendix, based on alliances among parties at the municipal level and switching voters between natione elections lead support to our classification of parties (TO BE ADDED) [19].

6.3 Data set

We collected data for a sample of local municipalities in Lombardia from 1988 to 2004. We chose 1988 as the starting year so as to make sure we had at least one observation on local elections for each municipality before the change in the electoral regime (which occurred in 1993). As we are basically interested to the effect of the threshold on political alliance and policy determination, we dropped observations from too small or too large municipalities, concentrating only on municipalities with a population size between 8,000 and 50,000 inhabitants (recall that the threshold is 15,000). (On occasion, we will focus on a smaller interval around the threshold, considering municipalities between 12,000 and 18,000 inhabitants). We were then left with 118 local municipalities below the threshold and 70 above the threshold. For each municipality we collected data on local elections (shares of votes for each party, voters turnout both at the first and the second ballot, votes collected by winning and losing candidates, coalition of parties, ex ante and ex post alliances, composition of the Giunta, party expressing the Mayor, etc.), as well as data on national elections[20] at the municipal level. As the

[19] Some of the current Italian parties did not actually exist at the beginning of our sample (e.g. Forza Italia, An, Margherita etc.), and some of the main parties which existed then have subsequently disappeared (e.g. Christian Democrats and Socialists). But based on the provenience of voters it is not difficult to assign the earlier party to our four group classification.

[20] Before the 1994 reform in national elections, the system was simply proportional, so that we collect the voting shares of parties at municipal levels for the national elections. After the 1994, the system for national elections became majoritarian, but 25% of the seats in the national parliament were still assigned according to a proportional rule and citizens also voted for parties. We then collected the votes for parties at municipal level

average duration of municipal governments is approximately 3,8 years[21], we then collected information for 732 municipal elections. We also build proxies to take into account when extremist and moderate parties were allied at the national level and when they were not, as this is an exogenous factor which may affect local alliances.

7 Some preliminary empirical results

We did not have the time to work through all our data set. So here we present just some preliminary evidence, supportive for the general theory we developed in the previous sections, leaving to future research the analysis of the remaining predictions. A crucial assumption of our model is that (some) voters are mobile between the two turns in a double ballot system; they vote for a candidate supported by a given coalition at the first turn, but they are prepared to vote again for a different candidate at the second ballot, if their original candidate does not pass the first turn. If this were not the case, all our theory would be meaningless, because it would mean that extremist candidates maintained all their bargaining power even in a dual ballot system (e.g. $\delta = 1$ in the model above). To check if this is true, we then examined voters' behavior in the sub-sample composed by only municipalities after the 1993 reform, above the 15,000 inhabitants threshold, and restricting further the analysis at those elections in which there was a second turn, that is, where no candidate was elected at the first turn.

Picture 1 plots the distribution of the sum of all votes to losing candidates at the first round of the two ballot system for this sub-sample, as a percentage of all eligible voters. The picture shows that vote for losers at the first round of the dual ballot system is substantial, ranging from 5% to almost 48% of all possible votes, with a median value around 25%. The question is then how many of these votes are retained at the second ballot, e.g. how many of the

(data for the lower chamber only).

[21] The legal duration of a municipality legislature is four years, but some of them fell before reaching the legal terminal period. Local governments and mayors changed more often before the 1993 electoral reforms, but usually this did not lead to new elections, but just to a change in government composition. As explained in the text, after the reform, the duration of the municipal government coincides with the duration of the municipal legislature.

voters voting for losing candidates at the first turn, vote again at the second turn for one of the two surviving candidate. To get a feeling about this, we compute for each municipality and each election the ratio of the difference between the sum of total votes at the second turn minus the votes of the two passing candidates at the first turn (at the numerator) divided by the votes for the losing candidates at the first turn (at the denominator). Thus, if the two passing candidates retained all their votes at the second ballot, and all voters for first ballot losers voted again at the second turn for one of these candidates, this ratio would be equal to 1; it would decline if only some of the first round losers voters voted again, and it would go to zero if the first two candidates just maintained their vote between turns. The ratio might actually become negative, if some of the first turn voters for winners decided for some reason not to vote again at the second. This index is of course not very precise, as we do not know the identity of voters (say, some voters who have not voted at the first turn may decide to turn out at the second one, and we cannot distinguish them and first round voters for losers). Still, picture 2, which plots the distibution of this index across the sub-sample, does suggest that a large number of voters for losers at the first turn vote again at the second one. The ratio is mostly positive, reaches 80% in some cases, and it has a median value of 40%[22]. Putting picture 1 and 2 together, we thus conclude that a substantial share of voters do vote for losers at the first turn, and many of these voters vote again for one of the surviving candidates at the second one, in line with the central assumptions of our model.

Turning now to predictions, Table 2 presents some evidence concerning the first of our theoretical results, the fact that at the first round of the two ballot system, more candidates or parties should be running than with respect to the single ballot. According to proposition , infact, if the electorate is polarized, under the single ballot system only two coalitions formed by extremist and moderate candidates should be viable in equilibrium, while according to propositions , depending on parameters, four candidates might run at the first round of the dual ballot system if there is no ex post bargaining, and four candidates should always run if there is ex post renegotiation (but recall our cautioning remarks on this point in section V). Our theory does not clearly distinguish between "candidates" and "parties", so that in

[22] In some cases, these second ballot voters may be faithful voters who vote for an endorsed candidate by their party, but as we are going to show below endorsement between ballots occurs rather rarely in our sample.

table 2 we present evidence for both parties and candidates (recall that in the Italian dual ballot system, several parties (=lists) may support the same candidate at the first round of the dual ballot, while at the single ballot a running candidate can only be supported by a single list). In table 2a we consider the number of parties. The table strongly supports the idea that there are more parties (=lists) running at the first round of the dual ballot system. Even controlling for population and population squared, the electoral system has a strongly significant positive effect on the number of parties running (on average there four parties more running at the dual ballot than in the single ballot), and this effect is still there, although reduced in size, when we only focus on the reduced sample of municipalities between 12,000 and 18,000 inhabitants (column 1 and 3). On the contrary, before the reform, while the number of parties was still positively correlated with population, the threshold itself had no separate effect (columns 3 and 4). Finally, column (5) shows that this conclusion is robust to the introduction of fixed municipalities effect for all the sample.

Table 2b repeats the same analysis with the number of candidates as dependent variable. Our theoretical results are again confirmed. After the reform, and above the threshold, there is in general one candidate more running for mayor than below the threshold. The effect is robust and statistically significant, and it is basically the same in the complete and in the reduced sample. The threshold itself had instead no effect before the reform. Finally, column (5) reports the result for all period, controlling for fixed municipalities effects. Here we get a contradictory result, in the sense that the number of candidates seem to fall after the reform and beyond the 15,000 threshold. But notice that we are here considering the entire sample, including the pre-reform period. In this period, the electoral system was proportional; citizens voted for parties in the municipal council, not for candidates, and the mayor was elected ex post, following the results of the elections. Thus, lacking a better alternative, we are here equating candidates to running parties for the proportional system. It is then not surprising that in this extended sample, the number of candidates above the threshold may fall with respect to the number of parties before the reform.

8 Concluding remarks

In this paper, we compare dual ballot and single ballot electoral rules. We argue that with a highly ideologically polarized electorate, dual ballot rules may be a useful device to reduce the infuence of small and extreme parties on policy. Some voters are mobile between rounds of elections, and this reduces the barganing power of extremist parties, which are no longer pivotal to win the elections. Thus, either less coalitions between extremists and moderates form under dual ballot rule, or if they form, they form on policy platforms which reflect more the interest of moderates. We study this effect by allowing or not allowing ex post bargaining and consider several extensions of the basic model. We plan to take these predictions to data, by exploiting a natural experiment which is offered by the Italian case. In Italy, mayors in municipalities below 15,000 inhabitants are elected according to a single ballot rule, mayors in municipalities above 15,000 inhabitants according to a dual ballot rule. We build a data set on a sample of municipalities in Lombardia which allows us to test our model. The preliminary results are encouraging.

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10 Appendix

Proof of Proposition 1 To formally prove Proposition 1, we need to compute the expected utilities of all parties in all possible party configurations. We need some extra notation. Let EV_i^P be the expected utility of party P under party configuration i, for $i = II, IIIa, IIIb, IV$, where: II refers to the two party configuration $(\{1,2\},\{3,4\})$, IV the four party configuration $(\{1\},\{2\},\{3\},\{4\})$, $IIIa$, the three party configuration $(\{1,2\},\{3\},\{4\})$; and $IIIb$,the three party configuration $(\{1\},\{2\},\{3,4\})$. These are the only possibile outcomes once the second stage of bargaining is reached.

4 parties $(\{1\},\{2\},\{3\},\{4\})$

Given assunption (A.1), the two extremist parties don't have a chance, and the election is won with probability $1/2$ by one of the two moderate parties. Hence, by (1), the parties expected utilities are:

$$\begin{aligned} EV_{IV}^1 &= EV_{IV}^4 = -\frac{\sigma}{2} \\ EV_{IV}^2 &= EV_{IV}^3 = -\sigma\lambda + \frac{R}{2} \end{aligned}$$

3 parties $(\{1\},\{2\},\{3,4\})$.

By assumption (A2), groups 3 and 4 together are larger than either group 2 or group 1 alone, for all realizations of η. Moreover, since $\lambda \geq 1/6$, voters in groups 3 and 4 always vote for the coalition $\{3,4\}$ for any policy $q \in [t^3, t^4]$, since the bliss point t^2 is farther away. This means that the coalition $\{3,4\}$ wins the election with certainty. Let $q^{34} \in [t^3, t^4]$ be the policy platform proposed by this winning coalition. Expected utility for the four parties then is:

$$\begin{aligned} EV^1_{IIIb} &= -\sigma q^{34} \\ EV^2_{IIIb} &= -\sigma(q^{34} - \frac{1}{2} + \lambda) \\ EV^3_{IIIb} &= -\sigma(q^{34} - \frac{1}{2} - \lambda) + \frac{R}{2} \\ EV^4_{IIIb} &= -\sigma(1 - q^{34}) + \frac{R}{2} \end{aligned} \tag{8}$$

The other three party outcome $(\{1,2\},\{3\},\{4\})$ is symmetric to this one and can easily be computed

2 parties $(\{1,2\},\{3,4\})$.

If both coalitions form, each coalition wins with probability $\frac{1}{2}$. The equilibrium payoffs for the 4 parties depends on which policy is agreed upon in each coalition. Suppose the two extremist parties, 1 and 4, are selected as agenda setters in their respective coalitions and suppose that they propose to their respective coalition partners their own bliss points, $t^1 = 0$ and $t^4 = 1$ respectively. The expected utility of each party in this case is:

$$EV^1_{II} = EV^4_{II} = EV^2_{II} = EV^3_{II} = -\frac{\sigma}{2} + \frac{R}{4} \tag{9}$$

Any more moderate policy can only increase the expected utility of the moderate parties in each coalition.

Comparing EV^2_{II} in (9) with EV^2_{IIIb} in (8), it is easy to see that $EV^2_{II} > EV^2_{IIIb}$ for any $q^{34} \in [t^3, t^4]$. In words, given that the coalition $\{3,4\}$ has formed, moderate party 2 is always better off forming a coalition with 1, irrespective of the policy q^{34} chosen by the opposing coalition, and even if its own coalition chooses party 1's bliss point as a policy. A similar comparison establishes that party1 is also better off forming a coalition with 2, irrespective of the value of $q^{34} \in [t^3, t^4]$ and even if the $\{1,2\}$ coalition chooses party

2's bliss point as a policy. Hence, forming the coalition $\{1,2\}$ is always a best response to the coalition $\{3,4\}$. Finally, going through similar computations, it is easy to verify that both 1 and 2 are always better off in forming the $\{1,2\}$ coalition, irrespective of which policy they agree upon, given that 3 and 4 are running alone. Thus, the two party configuration $(\{1,2\},\{3,4\})$ is the unique equilibrium outcome.

Since all parties are strictly better off inside their respective coalition than outside, whoever happens to be the agenda setter always proposes his own bliss point and this proposal is always accepted.

QED

Proof of Proposition 3 Suppose that the second stage of bargaining is reached. Extremist candidates are always better off in a two party system, since if they run alone they have no chances of winning. The issue is whether moderate candidates prefer to merge with the extremists or not, and on what policy platform.

Moderates as agenda setters Suppose first that the moderate candidates are the agenda setter inside each prospective coalition. Consider candidate 2, given that 3 and 4 have merged. If candidate 2 runs alone, as explained in the text, he wins with probability $1-h$. If he wins, he implements his bliss point and enjoys the rents from office, R. If he loses, he gets no rents and the policy implemented is $t^3 = 1/2 + \lambda$. Hence, using the same notation as in the proof of Proposition 1, candidate 2's expected utility when running alone and given that 3 and 4 have merged is:

$$EV^2_{IIIb} = (\frac{1}{2} - h)R - 2\sigma\lambda(\frac{1}{2} + h)$$

If instead candidate 2 merges with 1 and implements its preferred policy, then their party wins with probability $1/2$, but then candidate 2 has to share the rents from office with the other party member. Hence, candidate 2's expected utility when he merges with 1, given that 3 and 4 have merged is:

$$EV^2_{II} = (\frac{1}{4})R - \sigma\lambda$$

Comparing these two expressions, we see that 2 is indifferent between these two options if

$$h = \underline{H} \equiv \frac{R}{4(2\sigma\lambda + R)} \tag{10}$$

Hence, if $h < \underline{H}$, candidate 2 prefers to run alone, given that 3 and 4 have merged, while if $h > \underline{H}$, candidate 2 prefers to merge, given that 3 and 4 have merged.

Next, consider candidate 2's alternatives if candidates 3 and 4 do not merge. If 2 also runs alone, he wins with probability 1/2 and his expected utility is:

$$EV_{IV}^2 = -\sigma\lambda + \frac{R}{2} \tag{11}$$

If instead candidate 2 merges with 1 and is the agenda setter inside his coalition, given that 3 and 4 have not merged, than party $\{1, 2\}$ wins with probability $(1 + h)$ and candidate 2's expected utility is:

$$EV_{IIIa}^2 = (\frac{1}{2} + h)\frac{R}{2} - 2\sigma\lambda(\frac{1}{2} - h)$$

Comparing the last two expressions, we see that 2 is indifferent between these two options if

$$h = \bar{H} \equiv \frac{R}{4(2\sigma\lambda + R/2)} \tag{12}$$

For $h < \bar{H}$, candidate 2 prefers to run alone, given that 3 and 4 have not merged; while for $h > \bar{H}$, 2 prefers to merge with 1, given that 3 and 4 have not merged and that 2 is the agenda setter.

Comparing (10) and (12), we see that $\bar{H} > \underline{H}$. This makes sense: running alone is more attractive (i.e., the threshold of indifference is higher) if the opponents are also running alone. Hence, three cases are possible, depending on parameter values:

If $h < \underline{H}$, the handicap from running alone is so small that both moderate candidates always prefer *not* to merge with the extremists. In this case, if the second stage of bargaining is reached and the moderate candidates are drawn to be agenda setters, the equilibrium is unique and we have a four party system.

If $h > \bar{H}$, the handicap from running alone is so large that both moderate candidates always prefer to merge with the extremists. In this case, if the second stage of bargaining is reached and the moderate candidates are agenda setters, the equilibrium is again unique, and we have a two party system on the moderates' policy platforms.

Finally, if $\underline{H} \leq h \leq \bar{H}$, then multiple equilibria are possible, given that the second stage of bargaining is reached and the moderate candidates are agenda setters. Depending on the players' expectations about what the other candidates are doing, we could have both a two party or a four party system.

In all these cases, the policy platforms inside the coalitions coincide with those of the moderate candidates since the extremists are always willing to merge.

Extremists as agenda setters Next, suppose that extremist candidates are the agenda setters. Let $q^{34} \in [1/2+\lambda, 1]$ denote the policy proposal for party $\{3,4\}$ and $q^{12} \in [0, 1/2-\lambda]$ the policy proposal for party $\{1,2\}$. These policies need not coincide with the extremist candidates bliss points, since the extremists may have to deviate from their bliss points to get their proposals accepted. Our goal is to establish conditions under which such proposals might or might not be accepted by the moderate candidates. Again, we focus attention on candidate 2, under different expectations about what happens in the opposing party, since the extremists are alway better off when they merge.

Suppose that candidate 2 expects party $\{3,4\}$ to be formed on the policy platform q^{34}. Going through the same steps as above, candidate 2's expected utility if he rejects or accepts candidate 1's proposal of a platform q^{12} are respectively:

$$EV^2_{IIIb} = (\frac{1}{2} - h)R - \sigma(\frac{1}{2} + h)(q^{34} - \frac{1}{2} + \lambda)$$

$$EV^2_{II} = (\frac{1}{4})R + \frac{\sigma}{2}(q^{12} - q^{34})$$

Hence, candidate 2 is indifferent between these two alternatives for:

$$h = H(q^{12}, q^{34}) \equiv \frac{\sigma(\frac{1}{2} - \lambda - q^{12}) + R/2}{2\sigma(q^{34} - \frac{1}{2} + \lambda) + 2R} \tag{13}$$

Thus, if candidate 2 expect coalition 3,4 to be formed, he prefers to run alone (to merge) if $h < H(q^{12}, q^{34})$ (if $h > H(q^{12}, q^{34})$). Note that $H(.)$ is strictly decreasing in both arguments. Intuitively, as q^{12} increases it approaches candidate's 2 bliss point and the merger becomes more attractive; while as q^{34} increases it gets further away from candidate's 2 bliss point, and this too makes the merger more attractive for candidate 2 (since losing the election would cause more disutility).

By symmetry, if two parties are formed, in equilibrium the policy platforms agreed upon by each coalition must have the same distance from 1/2. Hence, $H(q^{12}, q^{34})$ can be rewritten (with a slight abuse of notation) as:

$$H^M(q) \equiv \frac{\sigma(\frac{1}{2} - \lambda - q) + R/2}{2\sigma(\frac{1}{2} + \lambda - q) + 2R} \tag{14}$$

for $q \in [0, 1/2 - \lambda]$ and where the M superscript serves as a reminder that 2 expects his opponents to merge. It is easy to see that $\underline{H} \leq H^M(q)$ for any $q \in [0, 1/2 - \lambda]$, where the first inequality is strict if $q < 1/2 - \lambda$ and it holds with the equal sign at the point $q = 1/2 - \lambda$. Moreover, $H_q^M(q) < 0$. Thus, the function $H^M(q)$ reaches a maximum at $q = 0$, where

$$H^M(0) = \frac{\sigma(\frac{1}{2} - \lambda) + R/2}{2\sigma(\frac{1}{2} + \lambda) + 2R}$$

The policy $q = 0$ is the point of most extreme symmetric extremism; at this choice, q^{12} and q^{34} coincide with the extremist candidates bliss points, 0 and 1 respectively. In words, as the policy q approved inside each coalition becomes symmetrically more extreme, a merger becomes less attractive for the moderate candidates, given that they expect a symmetric merger to be formed by their opponent. Hence, they will be more willing to run alone and refuse the merger, even if they expect a merger to occur in the opposing coalition.

Suppose now that candidate 2 does not expect a merger to occur in coalition 3,4. If he runs alone, either himself or the other moderate party wins with probability $\frac{1}{2}$. Hence his expected utility is the same as in (11) above.

If he instead accepts the offer from candidate 1 to form a coalition at policy q^{12}, his expected utility, given the expectation that the coalition 3,4 will not form, is:

$$EV_{IIIa}^2 = (\frac{1}{2} + h)\frac{R}{2} - \sigma(\frac{1}{2} + h)(\frac{1}{2} - \lambda - q^{12}) - 2\sigma\lambda(\frac{1}{2} - h)$$

which is an increasing function of q^{12}. Candidate 2 will then be indifferent between accepting 1's offer or running alone, given his expectations on 3,4 , if:

$$h = H^A(q) \equiv \frac{\sigma(\frac{1}{2} - \lambda - q) + R/2}{2\sigma(q - \frac{1}{2} + 3\lambda) + R}$$

for $q \in [0, 1/2 - \lambda]$ and where the A superscript serves as a reminder that 2 expects his opponents to merge.Candidate 2 will then accept 1's offer if $h \geq H^A(q)$ and refuses it if $h < H^A(q)$. Clearly, $H_q^A(q) < 0$ and $\bar{H} \leq H^A(q)$, with equality at $q = \frac{1}{2} - \lambda$.

We are now ready to characterize the equilibrium if the extremists are agenda setters and stage two of bargaining is reached. Specifically:

If $h < \underline{H}$, then there is no feasible offer by an extremist that can induce a moderate candidate to merge with him, whatever the moderate's expectations about the other coalition. This can be seen by noting that, as discussed above, $\underline{H} \leq H^M(q)$, $H^A(q)$ for all $q \in [0, 1/2 - \lambda]$. Hence, the unique equilibrium is a 4 party system with all candidates running alone.

If $h > \bar{H}$, then the moderate candidate, say candidate 2, always prefers to merge with the extremist on at least some (though not necessarily all) feasible policy platforms, whatever his expectations on the other coalition's behaviour. This can be seen by noting that $H^M(q) \leq \bar{H}$ for at least some $q \in [0, 1/2 - \lambda]$, and $H^A(q) = \bar{H}$ at the point $q = 1/2 - \lambda$. By symmetry, candidate 2 will rationally expect that the other coalition will always be formed. He would then accept any offer q by candidate 1 such that $h \geq H^M(q)$. Hence, the unique equilibrium is a two party system with a merger between extremists and moderates taking place on both sides.

The extremists candidates who act as agenda setters will then impose the policy platforms closest to their bliss points, subject to getting their proposal accepted. Since $H^M(0) \lesseqgtr \bar{H}$, the equilibrium platform in this case varies with the value of h. If $h \geq H^M(0)$, then both coalitions will form on the extremist candidates bliss points, 0 and 1 for coalitions $\{1, 2\}$ and $\{3, 4\}$ respectively. If $h < H^M(0)$, then coalition $\{1, 2\}$ will form on the policy $q^* \in [0, 1/2 - \lambda]$ such that $h = H^M(q^*)$, while coalition $\{3, 4\}$ will form on the symmetric policy $1 - q^*$. This can seen by noting that any policy $q' < q^*$ would not be accepted by candidate 2 (since by (13) $h < H(q', q^*)$), and any policy $q'' > q^*$ would be accepted by candidate 2 (since by (13) $h > H(q'', q^*)$) but suboptimal for candidate 1 who is the agenda setter. Since $H_q^M(q) < 0$, we have that $\frac{\partial q^*}{\partial h} = \frac{1}{H_q^M} \leq 0$, with strict inequality if $h < H^M(0)$. Thus, as h rises the equilibrium policy falls towards the extremists bliss point (or it remains constant if it is already at the extremist's bliss point).

Finally, if $\underline{H} \leq h \leq \bar{H}$, then two equilibrium outcomes are possible in pure strategies. (i) If the moderate candidate expects his moderate opponent to run alone, he also prefers to run alone (since $h \leq \bar{H} \leq H^A(q)$). Hence we have a four party equilibrium.(ii) If the moderate candidate expects his opponents to merge, then he also prefers to merge rather than running alone (since $\underline{H} = H^M(1/2-\lambda) \leq H^M(q) \leq h$ for at least some q). Going through the argument in previous paragraph, the equilibrium policy platform in this case coincides with the extremist's bliss point if $h \geq H^M(0)$, and it is q^* such that $h = H^M(q^*)$ if $h < H^M(0)$. (Again, recall that $H^M(0) \lesseqqgtr \bar{H}$, depending on paramter values).QED

Proof of Proposition 5 Suppose that neither moderate candidate has been endorsed. Then the probability that 2 wins is given by (5) and 2's expected utility is:

$$(\frac{1}{2}+\frac{\varepsilon_1}{e})R - 2\sigma\lambda(\frac{1}{2}-\frac{\varepsilon_1}{e})$$

If instead candidate 2 has been endorsed while candidate 3 has not, then the probability that 2 wins is given by (7) and 2's expected utility is:

$$(\frac{1}{2}+\frac{\varepsilon_1}{e}+\frac{\delta\alpha}{2e})\frac{R}{2} - 2\sigma\lambda(\frac{1}{2}-\frac{\varepsilon_1}{e}-\frac{\delta\alpha}{2e})$$

provided that the first expression in brackets is strictly less than 1 and the second expression in brackets is stricly positive, which occurs if $\varepsilon_1 \leq \frac{e}{2} - \frac{\delta\alpha}{2}$. If instead $\varepsilon_1 > \frac{e}{2} - \frac{\delta\alpha}{2}$, then the probability that 2 wins is 1 and his expected utility reduces to $R/2$.[23]

Candidate 2 is indifferent between these two alternatives if:

$$\varepsilon_1 = \breve{\varepsilon} \equiv \frac{\delta\alpha}{2}(1+\frac{4\sigma\lambda}{R}) - \frac{e}{2} \tag{15}$$

If $\varepsilon_1 > \breve{\varepsilon}$ then candidate 2 strictly prefers no endorsement, given that 3 has not been endorsed. While if $\varepsilon_1 < \breve{\varepsilon}$ then candidate 2 strictly prefers to be endorsed, given that 3 has not been endorsed.

Next, suppose that both moderate candidates have been endorsed by the extremists. Then the probability that 2 wins is given by (5), and 2's expected utility is:

$$(\frac{1}{2}+\frac{\varepsilon_1}{e})\frac{R}{2} - 2\sigma\lambda(\frac{1}{2}-\frac{\varepsilon_1}{e}) \tag{16}$$

[23] Assumption (4) implies that the first expression in brackets is always positive and the second one is always less than 1.

Suppose that 3 has been endorsed by 4, while 2 has not been endorsed. Then the probability that 2 wins is given by (6), and 2's expected utility is:

$$(\frac{1}{2}+\frac{\varepsilon_1}{e}-\frac{\delta\underline{\alpha}}{2e})R-2\sigma\lambda(\frac{1}{2}-\frac{\varepsilon_1}{e}+\frac{\delta\underline{\alpha}}{2e})$$

provided that the first expression in brackets is strictly positive and the second expression in brackets is stricly less than 1, which occurs if $\varepsilon_1 \geq \frac{\delta\underline{\alpha}}{2}-\frac{e}{2}$. If instead $\varepsilon_1 < -\frac{e}{2}+\frac{\delta\underline{\alpha}}{2}$, then the probability that 2 wins is 0 and his expected utility reduces to $-2\sigma\lambda$.[24]

Candidate 2 is then indifferent between these two options if

$$\varepsilon_1 = \check{\varepsilon}+\frac{\delta\underline{\alpha}}{2} \tag{17}$$

If $\varepsilon_1 > \check{\varepsilon}+\frac{\delta\underline{\alpha}}{2}$ then candidate 2 strictly prefers no endorsement, given that 3 has been endorsed. While if $\varepsilon_1 < \check{\varepsilon}+\frac{\delta\underline{\alpha}}{2}$ then candidate 2 strictly prefers to be endorsed, given that 3 has been endorsed.

By symmetry, 3 has similar preferences, but in the opposite direction and with respect to the symmetric thresholds $-\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2}$ and $-\check{\varepsilon}$ (eg. 3 prefers no endorsement, given that 2 has been endorsed, if $\varepsilon_1 < -\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2}$, and so on).

We are now ready to characterize the equilibrium. Suppose first that $\check{\varepsilon} > 0$. This then implies that $0 > -\check{\varepsilon}$. This equilibrium is illustrated in Figure 2. If $\varepsilon_1 \in [-\check{\varepsilon}, \check{\varepsilon}]$, then both moderates find it optimal to seek the endorsement of the extremists, no matter what their opponent does. If $\varepsilon_1 \in (\check{\varepsilon}, \check{\varepsilon}+\frac{\delta\underline{\alpha}}{2}]$, then candidate 3 still finds it optimal to seek the endorsment of 4 no matter what 2 does; and given 3's behavior, 2 also finds it optimal to seek the endorsement of 1. The same conclusion holds, but with the roles of 2 and 3 reversed, if $\varepsilon_1 \in [-\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2}, -\check{\varepsilon})$. Finally, if $\varepsilon_1 > \check{\varepsilon}+\frac{\delta\underline{\alpha}}{2}$ then candidate 2 finds it optimal to seek no endorsement no matter what 3 does, while 3 finds it optimal to seek the endorsement of 4 no matter what 2 does (since a fortiori $\varepsilon_1 > -\check{\varepsilon}$). By the same argument, the roles of 2 and 3 are reversed if $\varepsilon_1 < -\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2}$.

Next suppose that $\check{\varepsilon}+\frac{\delta\underline{\alpha}}{2} < 0$. This then implies that $-\check{\varepsilon} > -\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2} > 0$. This equilibrium is illustrated in Figure 3. If $\varepsilon_1 \in [\check{\varepsilon}+\frac{\delta\underline{\alpha}}{2}, -\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2}]$, then both moderates find it optimal to seek no endorsement, no matter what their opponent does. If $\varepsilon_1 \in [-\check{\varepsilon}-\frac{\delta\underline{\alpha}}{2}, -\check{\varepsilon})$, then candidate 2 still finds it optimal

[24] By (4), the first expression in brackets is always strictly less than 1 and the second expression in brackets is always positive.

to seek no endorsment no matter what 3 does; and given 2's behavior, 3 also finds it optimal to seek no endorsement. The same conclusion holds, but with the roles of 2 and 3 reversed, if $\varepsilon_1 \in (\breve{\varepsilon}, \breve{\varepsilon} + \frac{\delta\alpha}{2}]$. Finally, if $\varepsilon_1 > -\breve{\varepsilon}$ then candidate 2 still finds it optimal to seek no endorsement no matter what 3 does (since a fortiori $\varepsilon_1 > \breve{\varepsilon} + \frac{\delta\alpha}{2}$), while 3 finds it optimal to seek the endorsement of 4 no matter what 2 does.

Finally, suppose that $\breve{\varepsilon} + \frac{\delta\alpha}{2} > 0 > \breve{\varepsilon}$. This then implies $-\breve{\varepsilon} - \frac{\delta\alpha}{2} < 0 < -\breve{\varepsilon}$. This equilibrium is illustrated in Figure 4. For $\varepsilon_1 > \breve{\varepsilon} + \frac{\delta\alpha}{2}$ candidate 2 finds it optimal not to be endorsed, no matter what 3 does, while 3 finds it optimal to seek the endorsement of 4 no matter what 2 does (since in this case $\breve{\varepsilon} + \frac{\delta\alpha}{2} > -\breve{\varepsilon}$). The same holds, but with the roles of 2 and 3 reversed, if $\varepsilon_1 < -\breve{\varepsilon} - \frac{\delta\alpha}{2}$. If $\varepsilon_1 \in (-\breve{\varepsilon}, \breve{\varepsilon} + \frac{\delta\alpha}{2}]$, then 3 still finds it optimal to be endorsed by 4 no matter what 2 does. And given 3's behavior, now 2 also finds it optimal to be endorsed. Again, the same holds, but with the roles of 2 and 3 reversed, if $\varepsilon_1 \in [-\breve{\varepsilon} - \frac{\delta\alpha}{2}, \breve{\varepsilon})$. Finally, if $\varepsilon_1 \in [-\breve{\varepsilon}, \breve{\varepsilon}]$ then multiple equilibria are possible, since the optimal behavior of each moderate candidate depends on what his moderate opponent does. Hence, in equilibrium both seek the endorsement of their extremist neighbor or none of them does.

QED

Proof of proposition A1$\lambda < 1/6$ Suppose $\lambda < 1/6$ and consider stage 2 of our bargaining game in the single ballot case.

(i) If the moderates are the agenda setters, nothing changes with respect to our previous results in section 3. Extremists have no chances of winning if they run alone, and this will lead them to accept any offer from the moderates. Consequently, moderates will simultaneously propose their bliss point and this offer will be accepted by the extremists.

(ii) Things change when extremists are the agenda setters. To see this, let us decompose stage 2 of our bargaining game in its two component subgames: at step 1, simultaneoulsy, candidate 1 proposes q^{12} to candidate 2, and candidate 4 proposes q^{34} to candidate 3; at step 2, simultaneoulsy, candidates 2 and 3 decide if saying "yes" to the offer of their closer extremist and form a coalition, or if saying "no" and run alone. That is, stage 2 of our bargaining game can be thought of as a dynamic game in complete information, where player 1' s strategy set is any $q^{12} \in \left[0; \frac{1}{2} - \lambda\right]$, player 4's strategy set is any $q^{34} \in \left[\frac{1}{2} + \lambda; 1\right]$, players 2 and 3 strategy sets are either "yes" or "no", and where decisions are taken sequentially, with players 1 and 4 making their choices simultaneously at step 1, and players 2 and 3 making simultaneously their choices at step 2. We look for the subgame perfect equilibria of this game. We solve the game by working backwards.

Consider then step 2 of the game. Take first candidate 2. Suppose first that candidate 2 expects candidate 3 to say "yes" to the offer made by candidate 4. Then, it is clear that candidate 2 will say "yes" to the offer made by his extremist, only if the two following inequalities are met: $q^{34} \leq \frac{1}{2} + 3\lambda$ and $q^{12} \geq 1 - 2\lambda - q^{34}$, and will say "no" otherwise. The reason is simple. If $q^{34} > \frac{1}{2} + 3\lambda$ the bliss point of voters of group 3 is closer to t^2 than to q^{34}; then, under our assumption that voters vote sincerously, if candidate 2 said "no" and run alone, he would get all the votes of group 3 and win the elections for sure. By the same token, if $q^{12} < 1 - 2\lambda - q^{34}$, candidate 2 will never accept the offer made by candidate 1, because otherwise all voters belonging to group 2 will prefer to vote for coalition $(3, 4)$ instead and 2 would lose the elections for sure. If $q^{34} = \frac{1}{2} + 3\lambda$ and $q^{12} = 1 - 2\lambda - q^{34}$ voters of the two moderate groups are indifferent; for simplicity, we assume here that they vote however for the coalition that contains "their"candidate (alternatively, if $q^{12} = 1-2\lambda-q^{34}$, candidate 1 could always win the alleance of 2 by proposing $q^{12} = 1-2\lambda-q^{34}+\varepsilon$, where ε is any small number > 0). On the other hand, if candidate 2 expects candidate 3 to say "no", he will also refuse any offer by candidate 1, unless $q^{12} \geq \frac{1}{2} - 2\lambda$, because otherwise he would be better off in expected terms by running alone. In fact, given that candidate 3 has said "no", candidate 2 gets the same expected rents $\frac{R}{2}$ either if he says "yes" or if he says "no", so that his choice depends only on the policy proposal made by candidate 1. If candidate 2 runs alone, given that 3 runs alone, his expected loss from policy is $-\sigma\lambda$; and if he joins in a coalition with 1, his expected loss is $-\sigma(\frac{1}{2} - \lambda - q^{12})$. Comparing the two, we obtain that candidate 2 says "yes", if candidate 3 says "no", iff $q^{12} \geq \frac{1}{2} - 2\lambda$ (again, assuming that candidate 2 accepts 1's offer when indifferent). Repeating the same argument for candidate 3, we immediately obtain that 3, if he expects candidate 2 to say "yes", will say "yes" iff $q^{12} \geq \frac{1}{2} - 3\lambda$ and $q^{34} \leq 1 + 2\lambda - q^{12}$ and "no" otherwise; while if 3 expects candidate 2 to say "no", he will say "yes" iff $q^{34} \leq \frac{1}{2} + 2\lambda$ and "no" otherwise.

We can then compute the Nash equilibria in the subgame 2 for any possible vector of policy proposals (q^{12}, q^{34}).In particular:

Case 1. Suppose $q^{34} > \frac{1}{2} + 3\lambda$ and $q^{12} < \frac{1}{2} - 2\lambda$. Then, no matter what 3 does, candidate 2 will always say "no" to candidate 1's offer, and given that candidate 2 has said "no", 3's best reply is to say "no" too. Then, the only

Nash equilibrium of the subgame in this case is (No, No). By simmetry, the same is true if $q^{12} < \frac{1}{2} - 3\lambda$ and $q^{34} > \frac{1}{2} + 2\lambda$.

Case 2. Suppose $q^{34} > \frac{1}{2} + 3\lambda$ and $q^{12} \geq \frac{1}{2} - 2\lambda$. Then, candidate 2 says "no" if he expects 3 to say "yes" and says "yes" if he expects candidate 3 to say "no". In turn, candidate 3 says "yes" if he expects candidate 2 to say "yes" and $q^{34} \leq 1 + 2\lambda - q^{12}$; says "no" if he expects candidate 2 to say "yes" and $q^{34} > 1 + 2\lambda - q^{12}$. Candidate 3 says "no" if he expects candidate 2 to say "no". Thus, there are clearly two subcases to consider here.

2.1 If $q^{34} > 1 + 2\lambda - q^{12}$ (which is certainly the case if $q^{34} > \frac{1}{2} + 4\lambda$, given that $q^{12} \geq \frac{1}{2} - 2\lambda$), candidate 3 always says "no", no matter what 2 does, and given that 3 has said "no", 2's best reply is to say "yes". The only equilibrium is then (Yes, No), where from now on the first entry in the bracket always refers to candidate 2's choice.

2.2 On the other hand, if $q^{34} \leq 1 + 2\lambda - q^{12}$, candidate 3 wants to do what 2 does, while candidate 2 wants to do the opposite of what 3 does. The only equilibrium is then a mixed strategy equilibrium. At this equilibrium, candidate 3 plays "yes" with probability $g = \frac{\sigma\left(\lambda - (\frac{1}{2} - \lambda - q^{12})\right)}{\left(\frac{3R}{4} - \sigma(\frac{1}{2} - 2\lambda - (\frac{q^{12}+q^{34}}{2}))\right)}$ and candidate 2 plays "yes" with probability $p = \frac{\sigma\lambda + \frac{R}{2}}{\left(\frac{3R}{4} + \sigma(\frac{1}{2} + 2\lambda - (\frac{q^{12}+q^{34}}{2}))\right)}$. Notice that given our restrictions on q^{34}, q^{12} in this case, both probabilities lie between 0 and 1.

By symmetry, the same type of equilibria emerges when $q^{12} < \frac{1}{2} - 3\lambda$ and $q^{34} \leq \frac{1}{2} + 2\lambda$. That is, there will be a mixed strategy equilibria when $q^{12} \geq 1 - 2\lambda - q^{34}$ and the equilibrium (No, Yes) when $q^{12} < 1 - 2\lambda - q^{34}$.

Case 3. Assume $\frac{1}{2} + 2\lambda < q^{34} \leq \frac{1}{2} + 3\lambda$.

3.1 Suppose first it is also the case that $q^{12} < \frac{1}{2} - 3\lambda$.Then candidate 2's best reply is to say "yes" if he expects 3 to say "yes" and "no" otherwise. But candidate 3 best strategy is always to say "no", whatever 2 says. So the only equilibrium in this case is (No,No). By simmetry, this is also the equilibrium if $\frac{1}{2} - 2\lambda > q^{12} \geq \frac{1}{2} - 3\lambda$ and $q^{34} > \frac{1}{2} + 3\lambda$.

3.2 Suppose then that $\frac{1}{2} - 2\lambda > q^{12} \geq \frac{1}{2} - 3\lambda$. Then there are multiple equilibria. Each candidate says "yes" if he expects the other to say "yes" and says "no" if he expects the other to say "no". Then, both (No,No) and (Yes,Yes) are possible equilibria.

3.3 Finally, suppose $q^{12} \geq \frac{1}{2} - 2\lambda$. Then, whatever candidate 3 says, 2's best reply is always to say "yes". But given that 2 says "yes", 3's best reply is also to say "yes", so that the only equilibrium is (Yes, Yes). By symmetry, the same equilibrium occurs if $\frac{1}{2} - 2\lambda > q^{12} \geq \frac{1}{2} - 3\lambda$ and $q^{34} \leq \frac{1}{2} + 2\lambda$.

Case 4. Finally suppose $q^{34} \leq \frac{1}{2} + 2\lambda$ and $q^{12} \geq \frac{1}{2} - 2\lambda$. Then each candidate would always say "yes" whatever the other does, so that the only equilibrium is (Yes, Yes).

Inspection of the four cases above (and their subcases and symmetric counterparts) show that we have exausted all possible cases, so that we have completely characterized the equilibria of the subgame between the two moderates for any possible combinations of (q^{12}, q^{34}).Let us then go back at step 1 and compute the subgame perfect Nash equilibria of the entire game. In order to do so, it is useful to compute first the best reply function of one of the extremist (the other's follows by symmetry). Consider then candidate 1. Again, there are several cases to consider.

Case 1. Suppose candidate 1 expects candidate 4 to make a proposal $q^{34} > \frac{1}{2} + 3\lambda$ to candidate 3. Then, provided that rents are large enough, candidate 1's best reply is to make an offer $q^{12} \geq \frac{1}{2} - 2\lambda$ to candidate 2. In this case in fact, with at least probability p, candidate 2 will respond by accepting 1's offer, while any other proposal will lead to an equilibrium in the subgame where candidate 2 refuses 1's proposal for sure. More precisely, if $q^{34} > \frac{1}{2} + 4\lambda$, candidate 2 will accept any offer such that $q^{12} \geq \frac{1}{2} - 2\lambda$ and the coalition (1,2) will win for sure, as candidate 3 will then certainly refuse 4's proposal. Then, the best option for candidate 1 is to propose $q^{12} = \frac{1}{2} - 2\lambda$, as this is the one closer to his bliss point. On the other hand, if $\frac{1}{2} + 4\lambda > q^{34} > \frac{1}{2} + 3\lambda$, by selecting a q^{12} in the interval $\left[\frac{1}{2} - \lambda; \frac{1}{2} - 2\lambda\right]$, candidate 1 can enforce the mixed strategy equilibrium we described above. However, as both p and g depend on q^{12}, candidate 1 will select the policy proposal which maximizes his expected utility taking into account this dependence. (This proposal must be strictly above $\frac{1}{2} - 2\lambda$, because at $q^{12} = \frac{1}{2} - 2\lambda$, $p = 0$; see case 2.2 above). At any rate, if rents are large enough, it is easy to check that for candidate 1 it is always better to have candidate 2 accept his offer with some positive probability rather than refusing it for sure. This implies that the best reply by 1 must lie in the interval $\frac{1}{2} - \lambda \leq q^{12} < \frac{1}{2} - 2\lambda$.

Case 2. Suppose next that candidate 1 expects candidate 4 to make an offer $\frac{1}{2} + 2\lambda < q^{34} \leq \frac{1}{2} + 3\lambda$ to candidate 3. Then, 1's best reply depends on his expectations on which of the two equilibria, (Yes, Yes) or (No,No) will result in the subgame among moderates if he replies with $q^{12} \in \left[\frac{1}{2} - 2\lambda; \frac{1}{2} - 3\lambda\right]$. On the one hand, if he expects this equilibrium to be (Yes, Yes), his best reply is to set $q^{12} = \frac{1}{2} - 3\lambda$. Infact, any $q^{12} < \frac{1}{2} - 3\lambda$ would entail a different equilibrium in the subgame where candidate 2 would certainly refuse 3's proposal, and $q^{12} = \frac{1}{2} - 3\lambda$ satisfies the constraint $q^{12} \geq 1 - 2\lambda - q^{34}$ for any q^{34} in the interval $\frac{1}{2} + 2\lambda < q^{34} \leq \frac{1}{2} + 3\lambda$. Furthermore, $q^{12} = \frac{1}{2} - 3\lambda$, among the feasible choices which would support the equilibrium (Yes, Yes), is the one which maximize 1's utility as it is the closest to his bliss point. On the other hand, if 1 expects the equilibrium (No,No) to result if he plays a q^{12} in the interval $\frac{1}{2} - 2\lambda > q^{12} \geq \frac{1}{2} - 3\lambda$, his best choice is then to set $q^{12} = \frac{1}{2} - 2\lambda$, because this is the policy proposal that, while still entailing a positive reaction by candidate 2, it is closer to 1's bliss point.

Case 3. Suppose next that candidate 1 expects candidate 4 to make an offer $\frac{1}{2} + \lambda \leq q^{34} \leq \frac{1}{2} + 2\lambda$ to candidate 3. Then, his best reply is to set $q^{12} = \frac{1}{2} - 3\lambda$, because, since 1 expects 3 to say "yes" to 4's proposal, he also knows, that at the equilibrium, 2 will always say "yes" to any proposals by 1 such that $q^{12} \leq \frac{1}{2} - 3\lambda$, and again, among the feasible choices which support a positive answer by 2, $q^{12} = \frac{1}{2} - 3\lambda$ is the one that maximizes 1's utility.

This characterized entirely 1's best reply function; 4's best reply function is just symmetric to 1's one. Using these results, we can then finally prove (ii) and (iii) of Proposition A1. To do this, just note that whatever candidate 4 does, the optimal reply by candidate 1 is always to reply with either $q^{12} \geq \frac{1}{2} - 2\lambda$ or with $q^{12} = \frac{1}{2} - 3\lambda$, depending on the cases (symmetrically, whatever candidate 1 proposes, the optimal reply by candidate 4 is to propose either $q^{34} \leq \frac{1}{2} + 2\lambda$ or $q^{34} = \frac{1}{2} + 3\lambda$). And, case (ii), $q^{12} = \frac{1}{2} - 3\lambda$ is the best reply to $q^{34} = \frac{1}{2} + 3\lambda$ and viceversa, if the extremists expect the moderates to reach in the subgame the equilibrium (Yes, Yes) when faced with these proposals. If moderates actually stick to this equilibrium in the subgame, $q^{12} = \frac{1}{2} - 3\lambda$, $q^{34} = \frac{1}{2} + 3\lambda$ is a subgame perfect equilibrium of the entire game, as all expectations are confirmed at the equilibrium, and each strategy by each player is optimal given each other player's strategy. On the other hand, case (iii), if extremists expect moderates to reach in the subgame the equilibrium (No, No) when faced with policy proposals $\frac{1}{2} + 2\lambda < q^{34} \leq \frac{1}{2} + 3\lambda$ and $\frac{1}{2} - 2\lambda > q^{12} \geq \frac{1}{2} - 3\lambda$, the best reply by candidate 1 if he expects 4 to play $q^{34} \leq \frac{1}{2} + 2\lambda$ is to set $q^{12} = \frac{1}{2} - 3\lambda$, and given that candidate 1 has chosen $q^{12} = \frac{1}{2} - 3\lambda$, the best reply by candidate 4 is to choose $q^{34} = \frac{1}{2} + 2\lambda$. Provided that expectations on the subgame equilibrium are confirmed at the equilibrium, this is then a subgame perfect equilibrium of the entire game. Repeating the argument, one can show that $q^{12} = \frac{1}{2} - 2\lambda$, $q^{34} = \frac{1}{2} + 3\lambda$ is also a subgame perfect equilibrium of the bergaining game, when extremists expect moderates to reach in the subgame the equilibrium (No, No) when faced with policy proposals $\frac{1}{2} + 2\lambda < q^{34} \leq \frac{1}{2} + 3\lambda$ and $\frac{1}{2} - 2\lambda > q^{12} \geq \frac{1}{2} - 3\lambda$.

QED

Add Figures 1-4

Figure 1: Distribution of voters' preferences.

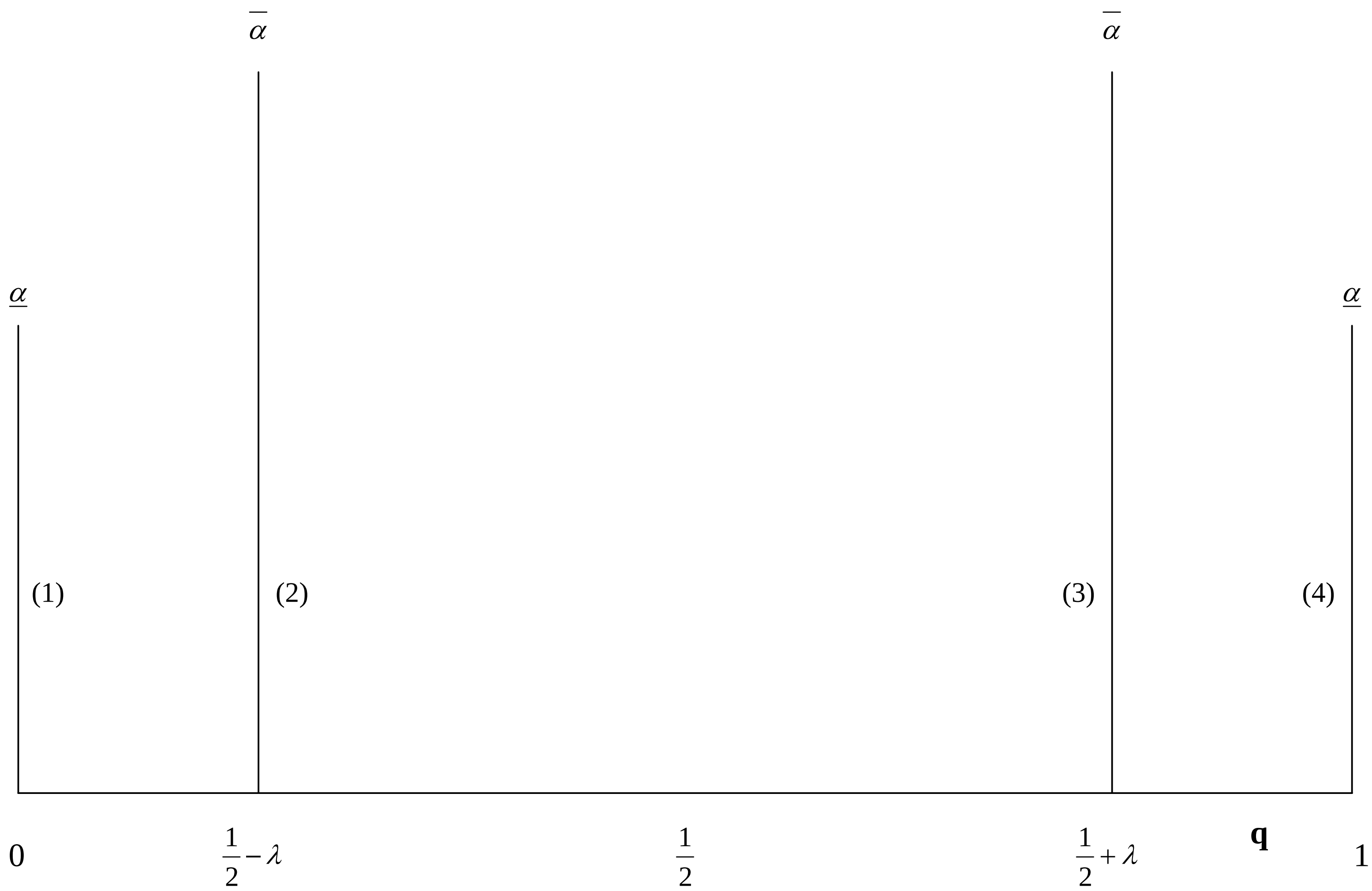


Figure 2

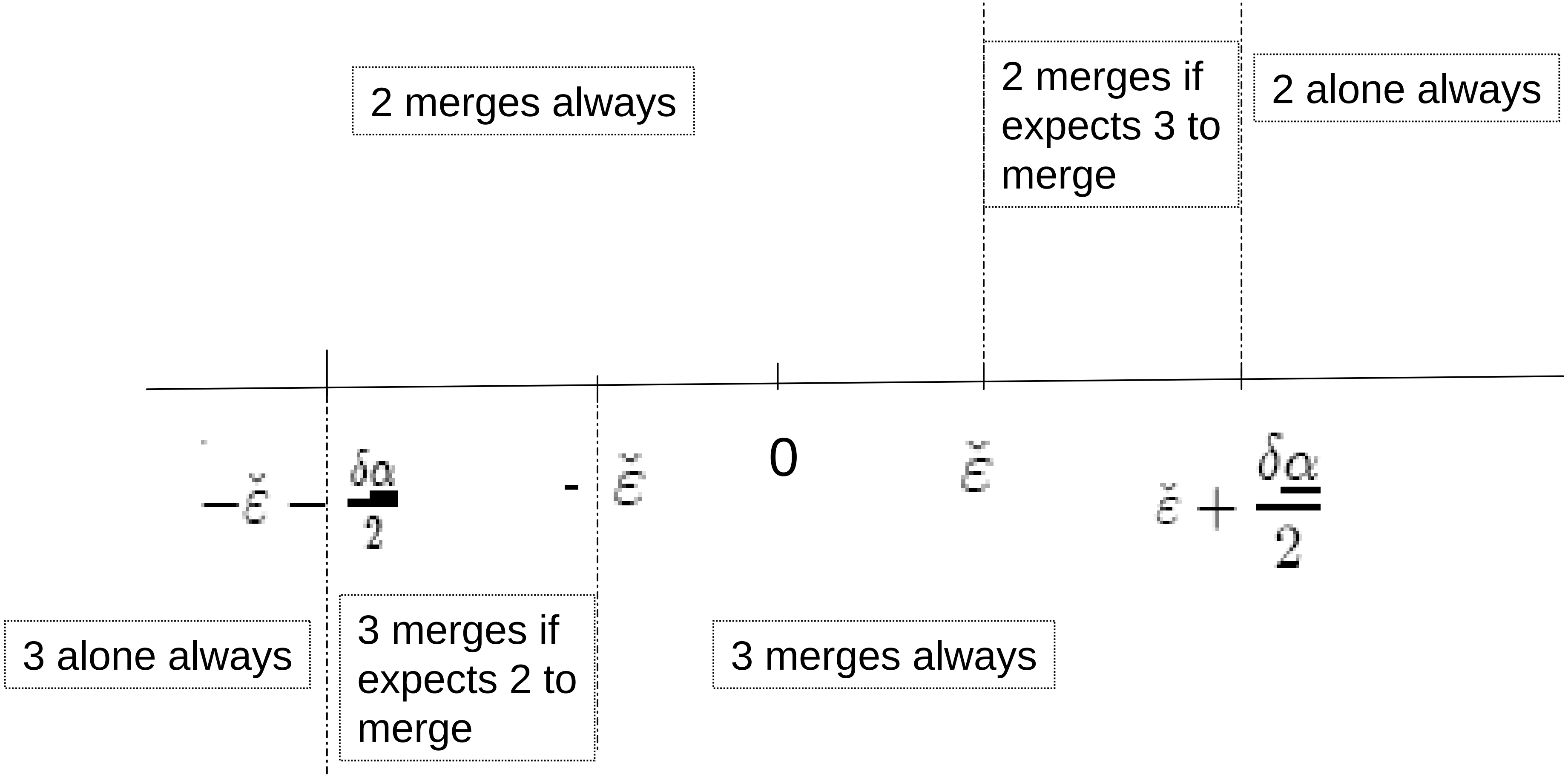

2 merges always
2 merges if expects 3 to merge
2 alone always
$-\breve{\varepsilon} - \frac{\delta\alpha}{2}$
$-\breve{\varepsilon}$
0
$\breve{\varepsilon}$
$\breve{\varepsilon} + \frac{\delta\alpha}{2}$
3 alone always
3 merges if expects 2 to merge
3 merges always

Figure 3

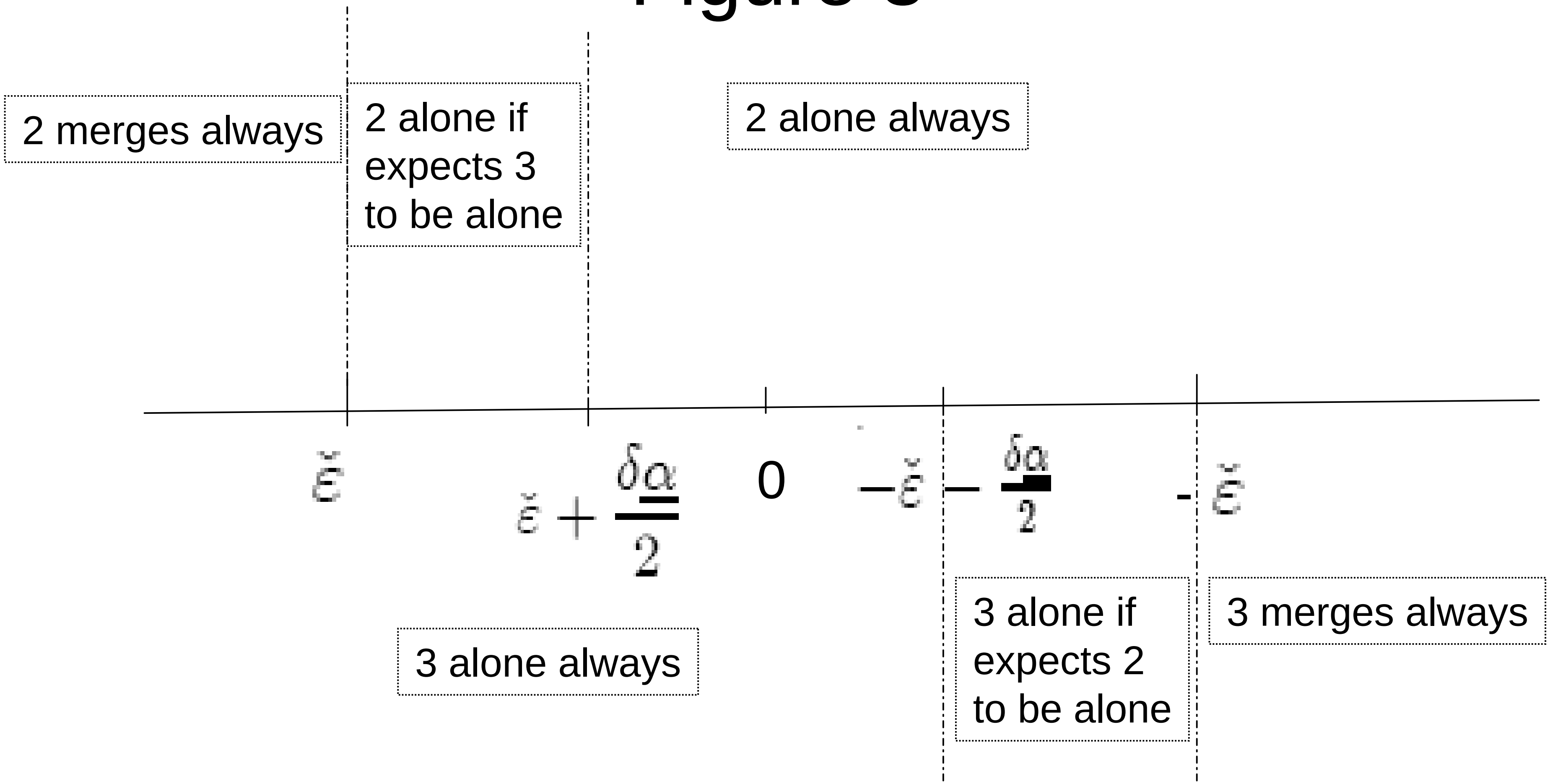
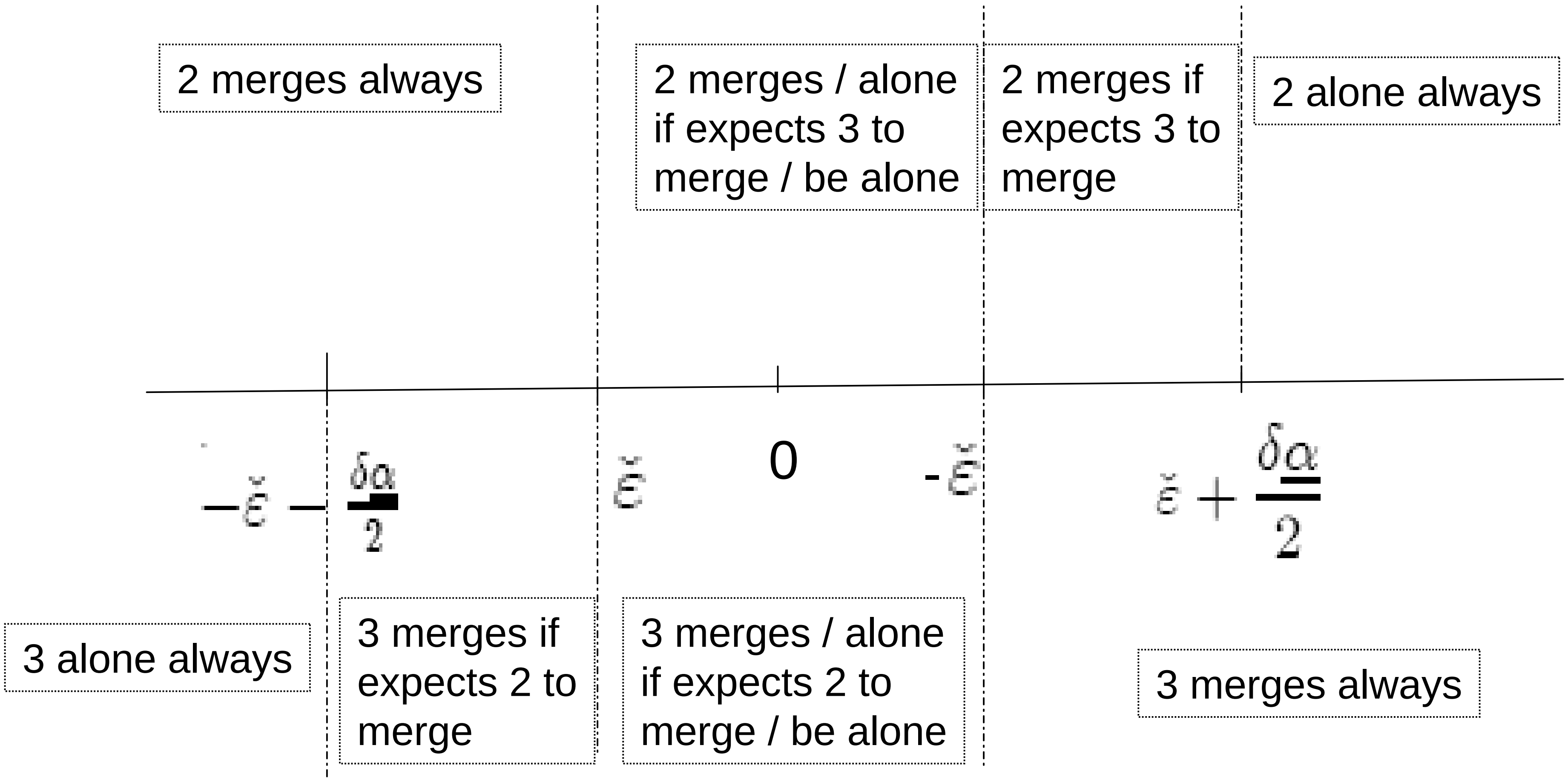
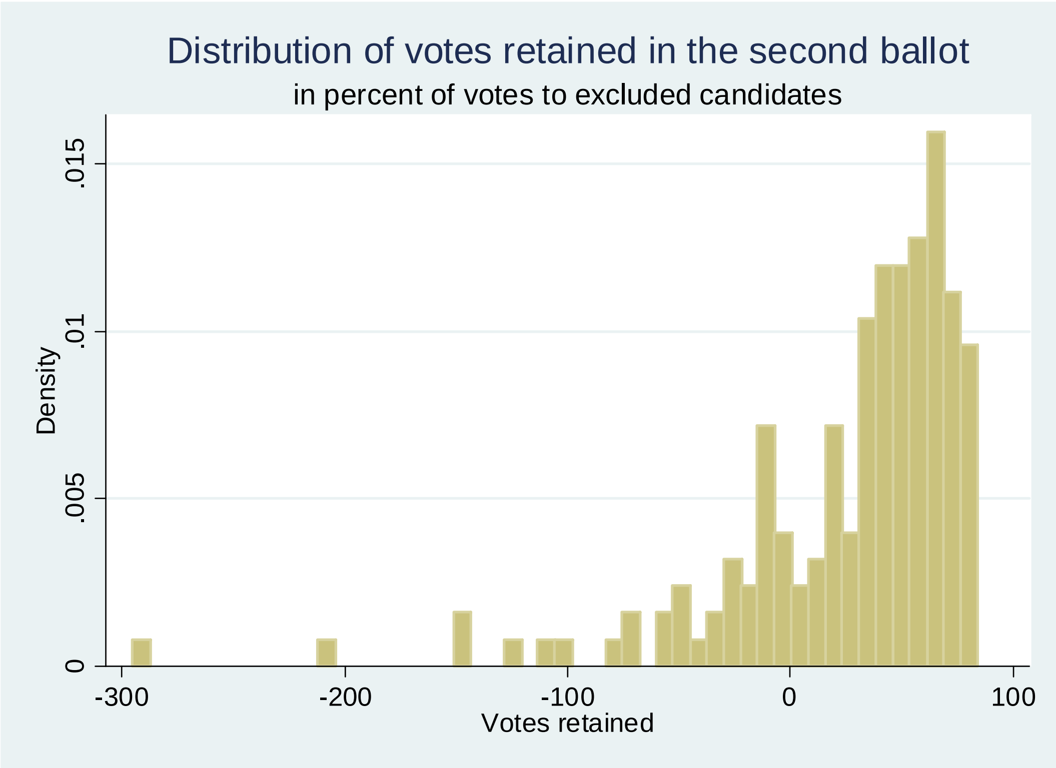
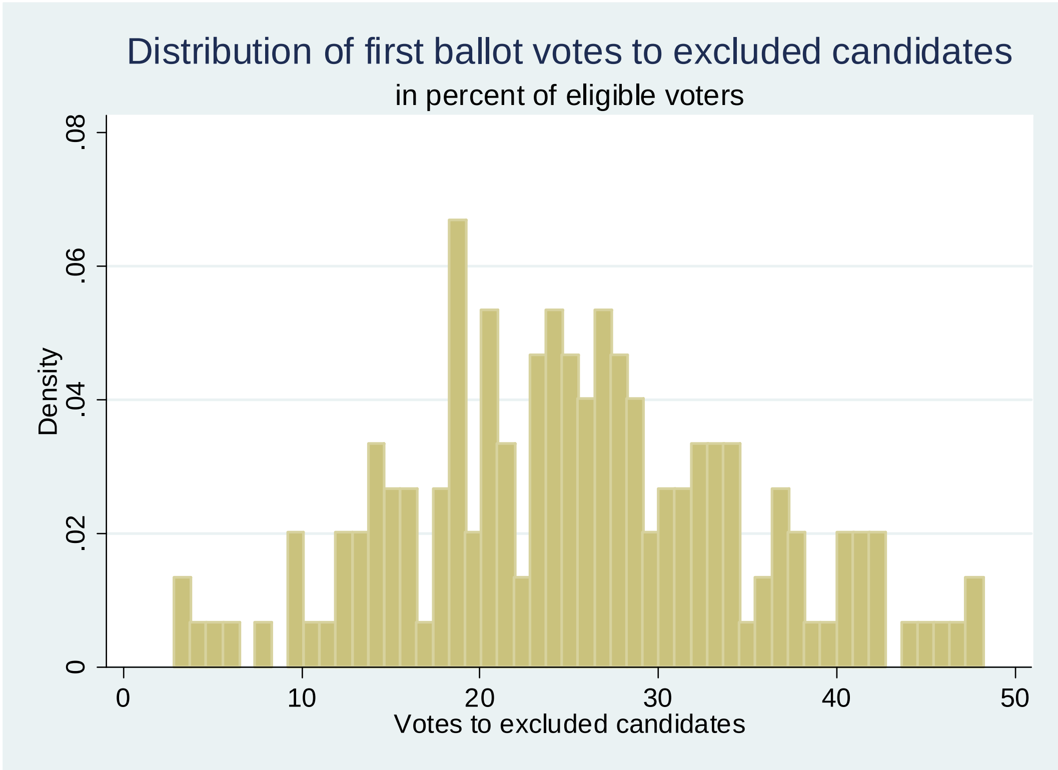

2 merges always
2 alone if expects 3 to be alone
2 alone always
$\breve{\varepsilon}$
$\breve{\varepsilon} + \frac{\delta\alpha}{2}$
0
$-\breve{\varepsilon} - \frac{\delta\alpha}{2}$
$-\breve{\varepsilon}$
3 alone always
3 alone if expects 2 to be alone
3 merges always

Figure 4


2 merges always
2 merges / alone if expects 3 to merge / be alone
2 merges if expects 3 to merge
2 alone always
$-\breve{\varepsilon}-\frac{\delta\alpha}{2}$
$\breve{\varepsilon}$
0
$-\breve{\varepsilon}$
$\breve{\varepsilon}+\frac{\delta\alpha}{2}$
3 alone always
3 merges if expects 2 to merge
3 merges / alone if expects 2 to merge / be alone
3 merges always



Distribution of votes retained in the second ballot
in percent of votes to excluded candidates
Density
0
.005
.01
.015
-300
-200
-100
0
100
Votes retained



Distribution of first ballot votes to excluded candidates
in percent of eligible voters
Density
0
.02
.04
.06
.08
0
10
20
30
40
50
Votes to excluded candidates

Table 1 – Candidates and Parties: Summary Statistics

	Pre-reform			*Post reform*		
Pop. Size	< 15K	> 15K	Δ	< 15K	> 15K	Δ
N. parties	6.83 (0.15)	8.74 (0.23)	1.91 (0.28)***	3.88 (0.06)	9.68 (0.19)	5.81 (0.20)***
N. candidates	6.83 (0.15)	8.74 (0.23)	1.91 (0.28)***	3.86 (0.58)	5.16 (0.11)	1.30 (0.12)***
Obs	120	73		320	219	

Standard errors in parenthesis, *** significant at 1%

Δ = Number of (.) in municipalities > 15K minus Number of (.) in municipalities < 15K

Table 2a: Electoral system and number of parties

	(1)	(2)	(3)	(4)	(5)
			Number of parties		
Dual ballot	4.30	1.87	-0.39	-0.01	0.72
	(0.38)***	(0.67)***	(0.46)	(0.88)	(0.26)***
Single ballot					-3.12
					(0.17)***
Population	1.73	-55.37	3.47	-4.76	6.54
	(0.62)***	(19.42)***	(0.82)***	(26.90)	(2.38)***
Pop. Squared	-0.00	0.00	-0.00	0.00	-0.00
	(0.00)	(0.00)***	(0.00)***	(0.00)	(0.00)
Fixed effects	Year	Year	Year	Year	Municipality
Population size	All	12K -18K	All	12K -18K	All
Years	Post-reform		Pre-reform		All years
Obs.	539	163	193	49	732
Adj. R2	0.71	0.64	0.44	0.07	0.35
N. municipal.					188

Robust standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%
Population is measured in 10000 (10K)
Year fixed effects included columns (1-4)
Municipality fixed effects included in column (5)
In columns (1-4) the variable *Dual ballot* equals 1 in the municipalities above 15K inhabitants and 0 otherwise
In column (5) the variable *Dual ballot* equals 1 in the municipalities above 15K inhabitants after 1992 and 0 otherwise, the variable *Single ballot* equals 1 in the municipalities below 15K inhabitants after 1992 and 0 otherwise

Table 2b: Electoral system and number of candidates

	(1)	(2)	(3)	(4)	(5)
	Number of candidates				
Dual ballot	0.86	1.06	-0.41	-0.01	-3.47
	(0.22)***	(0.41)***	(0.46)	(0.88)	(0.21)***
Single ballot					-3.07
					(0.15)***
Population	0.38	8.75	3.46	-4.76	1.59
	(0.43)	(11.05)	(0.82)***	(26.90)	(2.04)
Pop. Squared	-0.00	-0.00	-0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)***	(0.00)	(0.00)
Fixed effects	Year	Year	Year	Year	Municipality
Population size	All	12K -18K	All	12K -18K	All
Years	Post-reform		Pre-reform		All years
Obs.	539	163	193	49	732
Adj. R2	0.33	0.25	0.44	0.07	0.58
N. municipal.					188

Robust standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%
Population is measured in 10000 (10K)
Year fixed effects included columns (1-4)
Municipality fixed effects included in column (5)
In columns (1-4) the variable *Dual ballot* equals 1 in the municipalities above 15K inhabitants and 0 otherwise
In column (5) the variable *Dual ballot* equals 1 in the municipalities above 15K inhabitants after 1992 and 0 otherwise, the variable *Single ballot* equals 1 in the municipalities below 15K inhabitants after 1992 and 0 otherwise